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Methods of Analysis of Yield from Antecedent Winter Water

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METHODS OF ANALYSIS OF YIELD
FROM ANTECEDENT WINTER WATER

BY

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FROM ANTECEDENT WINTER WATER

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Doctor of Philosophy, and is acceptable as meeting the thesis requirements for this degree. Acceptance of this thesis does not imply that the conclusions reached by the candidate are necessarily the conclusions of the major department.

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LIST OF FREQUENTLY USED SYMBOLS

A = area (cm^2)

AFIA = amount of water applied as fall irrigation water (cm)

AI = amount of water contained within the root zone of a fall irrigated plot as measured the following spring (cm H_2O/depth of root zone)

ANI = amount of water contained within the root zone as measured the following spring of a plot having the same surface cover as the fall irrigated plot and the same moisture content prior to the time of fall irrigation but not fall irrigated (cm H_2O/ depth of root zone)

ASF = actual snowfall (cm as liquid water)

C = specific heat of vaporization (cm water/cal cm^2)

D = diffusivity (cm^2/day)

ESF = effective snowfall (cm as liquid water)

ET or ETa = actual evapotranspiration (cal/cm^2 day)

ET_p = potential evapotranspiration (cal/cm^2 day)

F = standard F, the ratio of variance from means over variance of individual

FC = field capacity (cm^3 water/cm^3)

G = heat flux density from ground (cal/cm^2 day)

G_i = relative growth during ith cycle

K = hydraulic conductivity (cm/day)

$K_{a,c,\text{ or }co}$ = crop coefficients

L = length (cm)

Mc = moisture content of profile (cm/120 cm)

MCC = multiple correlation coefficient

MD = moisture deficit

PAM = percent available moisture content

PL = percent of water loss in the profile of a fall irrigated plot compared with a nonirrigated plot (%)

PWP = permanent wilting point (cm^3 water/cm^3)

PY = potential yield (bushels, tons, kilograms, etc.)

Q = quantity of water (cm^3)

R = surface roughness factor (dimensionless)

R_n = net radiation (cal/cm^2 day)

S = standard deviation from mean

SE = standard error of Y with respect to partial regression coefficient of new variable in stepwise linear regression

SMC = actual soil moisture content (cm^3 water/cm^3)

T = temperature (°C)

U_z = horizontal wind speed at height z (km/day)

Y = total yield (bushels, tons, kilograms, etc.)

e_z = actual vapor pressure at height z (mb)

e_z^o = saturation vapor pressure at height z (mb)

h_t = total hydraulic potential (cm)

i = hydraulic gradient (cm/cm)

i (finite differences) = depth increments (cm)

j (finite differences) = time increments (days)

lsd = least significant difference

q = flux (cm^3/cm^2 day)

r^2 = coefficient of determination

t = time (days)

z = height above or below ground surface

θ = volumetric moisture content (cm^3 water/cm^3)

α = psychrometric constant (mb/°C)

Δ = slope of saturation vapor pressure curve de°/dT (mb/°C)

λ = a regression constant or weighing factor

Ψ = pressure head (cm)

** = significantly different at the .99 level

* = significantly different at the .95 level

† = significantly different at the .9 level

NS = not significantly different

METHODS OF ANALYSIS OF YIELD
FROM ANTECEDENT WINTER WATER

Abstract

Charles Gregg Carlson

Under the supervision of Dr. M. L. Horton

This investigation is an evaluation of methods of analysis of water conserved during off-season periods and evaluation of the effect on the subsequent crop. Energy and water shortages during the mid-summer peak irrigation season have forced farmers to consider the possibility of irrigation at times other than peak growth stages as an alternative practice. Dryland farmers are interested in the benefit of water conserving cultural practices such as summer fallow and snow trapping. To evaluate the benefit of these off-season practices, a yield function based upon water availability is necessary. In order to better understand the effects of off-season irrigation and other cultural practices, a theoretical analysis using the basic flow equation is beneficial.

In this study consideration was given to: (1) development of a method for determining the feasibility of fall irrigation and off-season cultural practices, (2) development of a yield function for corn based upon water availability at different physiological stages of growth, (3) development of a theoretical mathematical simulation procedure for examining moisture movement under field conditions.

Moisture profiles of irrigated and nonirrigated plots of corn, wheat and alfalfa were monitored over a winter and through a growing

season. Data collected were used to develop a function to predict the effective amount of spring carry-over from a fall irrigation. Data collected during the growing season were used to determine the response of the corn crop to several magnitudes of stress at different physiological stages of growth.

A mathematical simulation of a soil moisture profile under field conditions with simplified boundary conditions was performed. Verification of the simulation was accomplished using measured field water contents.

Results indicate that fall and/or early spring irrigation applied to a deep fine-textured soil can cause crop yield depression during years of greater than average fall, winter and early spring precipitation.

The simulation of field water movement resulted in soil water profiles that were adequate representations of field plots.

INTRODUCTION

Throughout the North Central Great Plains the drought conditions of the last few years combined with rapid inflation of land prices, increasing machinery expenses and rising production costs have brought intense new interest by farmers in irrigation. Irrigation agriculture appears to many farmers to be the only viable insurance policy which is capable of underwriting the uncertainty of the limited and sporadic rainfall of the Central Plains region.

As a result of current commodity surpluses, agriculture has a low priority for the allocation of energy and water resources. If this present trend continues, the result will be limited irrigation water and energy supplies and ultimately limited irrigation acreage. Even if irrigation is not affected by the priority allocation of water and energy, it will be greatly affected by the present rapid inflation of energy costs.

To cope with the increasing energy expense and decreasing energy and water supplies, irrigation agriculture needs to become more efficient and more knowledgeable in the conservation of both energy and water. It is here that the science of maximizing crop production while minimizing water application becomes significant.

There are many examples of situations where energy and water are available at one time of the year but are in short supply at other times. In the case of water, this is especially true when pumping from a river or stream which is full in the spring but nearly dry at other times. In the case of energy, the hottest portion of the summer is the period of maximum evapotranspiration and also corresponds with the peak energy utilization period for non-agricultural purposes. Besides these problems of timing, there exists the continual need to insure that at some time during the year excess amounts of water are applied to meet leaching requirements.

The objective of this study was to develop methods for analyzing the effectiveness of off-season irrigation or precipitation to subsequent crop production.

CHAPTER I

SECTION 1

INTRODUCTION AND LITERATURE REVIEW OF OVERWINTER WATER CONSERVATION

In semi-arid regions such as South Dakota, plant-available soil moisture during the growing season is frequently a limiting factor for both grain and forage production. Plant-available soil moisture during the growing season exists either as a result of in-season precipitation, irrigation or as antecedent moisture carried over from the non-vegetative growth portions of the season. The antecedent soil moisture may result from off-season precipitation, artifically applied fall irrigation, cultural practices such as summer fallowing, or the artificial snow accumulation dur to snow trapping structures.

Off-season water conserving practices, under specific circumstances, can be beneficial to the subsequent crop. What is needed is to quantitatively define the specific circumstances under which this can occur.

It is important, especially in localized areas of the Central Great Plains, that factors other than the conservation of water be considered. For example in regions of high salinity parent material, or high salinity irrigation water, consideration must be given to the leaching of excess salts from the root zone.

Agronomists have known for years that infiltration and redistribution rates for water are affected greatly by the amount of moisture that exists in the profile. Analytically stated, the hydraulic conductivity of moist soils is related to the volumetric moisture content in a logarithmic type relationship (Hillel, 1971). Therefore, the antecedent moisture content of a particular soil profile in the fall and the local weather conditions throughout the winter are the controlling parameters that influence the effectiveness of off-season irrigation and water conservation (Reid, 1975) and (Collis-George and Lal, 1971). In areas where precipitation probabilities indicate there will be more rainfall than the soil profile will hold, it is not likely that off-season irrigation or cultural practices which conserve or trap water could be economically justifiable. In the Northern Great Plains there exist the conditions of predictable winter precipitation being less water than is required to fill deep, fine textured soil profiles and a large percentage of this precipitation falls as snow which is vulnerable to being blown off the fields. With this situation, it is probable that off-season irrigation and/or cultural methods of conservation can be of some economic value.

Since alfalfa is a perennial crop, it has the potential of utilizing soil moisture from the profile for a longer period of time than do the grains and thus over the entire season use more moisture from deeper in the profile. For this reason, alfalfa is a crop that has a high probability of benefiting from fall and winter irrigation and conservation. Haas and Willis (1971), utilizing level benches to collect snow in a five-year study in North Dakota, found that they collected from 12.2 to 23.1 cm of water on the benches as compared to 3.6 cm of water on the slopes. Alfalfa production ranged from 7170 to 9590 kg/ha in the benches as compared to 3360 kg/ha on the untreated slope. They were convinced that this type of conservation practice could benefit farmers.

Wheat is another crop that could benefit from water conservation. In some early work from Canada, Staple and Lehane (1954) concluded that conserved moisture is only 68% as effective as rainfall in the growing season. Robertson (1974), in an analysis which spanned 50 years at Swift Current, Saskatchewan, concluded that preseason precipitation was the second most important factor contributing to the yield of spring wheat. Preseason precipitation accounted for 14% reduction in the coefficient of determination for yield or about 20% of the total reduction. Of over fifty factors considered, the only factor considered more important was June rainfall. Thompson (1969), in an analysis of data over six midwestern states from 1920 to 1968, determined that highest yields in North and South Dakota are significantly associated with years when they received 25.4 cm of precipitation or more during the period from August to March as compared to the usual 18.3 and 20.1 cm. It is quite interesting to note that states such as Illinois and Indiana obtained maximum yields in years of less than their normal 50.8 cm of precipitation during this August through March period. Thus, too much moisture in the off-season on wheat fields may result in yield decreases.

Corn production covers an extended growing season and one could speculate that corn may not be as sensitive to early spring moisture storage as is wheat. Holt et al. (1964) using three years of corn yield data at nine different locations in the northwestern corn belt found yield correlations of 50% in 1958, 55% in 1960, and 9% in 1961 which could be attributed to differences in soil moisture at planting. They concluded that the effect of stored soil moisture will be minimized by above-normal rainfall during the critical growth period of the corn plant. The positive correlations mentioned above by Holt et al. (1964) indicate that there is at least a possibility for obtaining a profit from off-season cultural practices which will ensure a full soil moisture profile at the beginning of the growing season. To evaluate the economic aspects of these cultural practices, it is necessary to study what factors are important in the determination of moisture gain or loss over the winter season. Several workers, (Holt and Timmons, 1968), (Mathews and Army, 1960), (Hobbs and Krogman, 1970), have documented a significant negative correlation between the initial water content in the fall and the amount of moisture gained over the winter. If the soil moisture profile is full, you can not add any more water, or, the drier the profile, the more moisture you will be able to store. Another logical conclusion is that the greater the precipitation the greater the quantity of water which has a chance of being absorbed by the soil profile. The physical state in which the moisture comes (snow, rain, sleet, flood, etc.), and the physical condition of the soil surface are important factors which must be considered as variables in an evaluation of off-season moisture storage.

Over 20 years of work at Swift Current, Saskatchewan, Staple et al. (1960) found that 37% of the overwinter precipitation was conserved on stubble land as compared to 9% on fallow land. The gain on fallow land was mainly a result of rainfall while the stubble collected considerable amounts of snow. Fall moisture reduced the overwinter conservation by about 0.51 cm for every 2.54 cm of stored moisture. Holt and Timmons (1968), working in western Minnesota and eastern South Dakota, found a reduction of about 1.02 cm of overwinter moisture conservation for every 2.54 cm of fall stored water. Ferguson et al. (1964), working in Montana, showed a loss of about 0.64 cm of conserved water for every additional 2.54 cm of water in the fall. Ferguson et al. data are somewhat unique in that two of their plots lost a portion of the stored moisture and, thus, had a negative net gain. Willis and Haas (1969) found that with a spring small grain and summer fallow system there is essentially no conservation of winter precipitation during the winter after summer fallow. Musick (1970) reported results from Bushland, Texas, which showed that a dry soil will conserve between 30-50% of fall and winter precipitation while a wet soil will conserve only about 10% of the precipitation. Hobbs and Krogman (1971), working in Alberta, Canada, recommend that fall irrigation should be practiced on the Canadian prairies only when the moisture in the root zone is less than half of the available water capacity. They conclude that for each 2.54 cm of available moisture in the profile in the fall, the overwinter water conservation will be reduced by about 1.17 cm. Mathews and Army (1960) in a rather complete summary of fallow crop production from 450 crop years of data at 25 locations throughout the Great Plains concluded that the average overwinter storage was 5.13 cm of moisture or 23% of the precipitation.

On an every other year basis, the average was 10.06 cm or 16.3% of the precipitation. This figure could go as high as 19.20 cm for a dry fall and wet winter and as low as -8.36 cm for a wet fall and dry winter. It is quite apparent that the variability is great.

CHAPTER I

SECTION 2

MATERIALS AND METHODS OF OVERWINTER WATER CONSERVATION

The work reported here was conducted on the James Valley Agricultural Research and Extension Center located about 9.6 km east of Redfield, S.D., on the NE 1/4 of Section 2, T116N, R63W in Spink County, S.D. Classification of the soil (Westin et al., 1954) is a Great Bend silt loam occurring on level positions in the southern part of the Glacial Lake Dakota Plain. A detailed description of the profile is given in Table I-1.

Soil fertility and fertilizer application rates were determined for each crop from soil samples taken throughout the plots and from tests conducted by the Soil Testing Laboratory at South Dakota State University. The recommended fertilizer for each crop was applied.

Soil moisture was monitored at 30 cm depth intervals (i.e. 15-45, 45-75, etc.) using a Troxler model 105A neutron probe and a Troxler model 200B scalar. The top 15 cm of the profile was sampled gravimetrically using an Oakfield hand probe. During frozen periods, samples at the surface were taken with a pick.

Weather data used in the analysis were obtained from several sources. Daily precipitation and min-max temperature data were taken directly from the Redfield weather station which is located near the office of the irrigation farm, about 400 meters from the plots. Wind run, total short-wave radiation, and dewpoint temperatures were taken from the nearest weather station at Aberdeen, S.D. which is about

Table I-1. Detailed profile description of the Great Bend silt loam soil.

Location: James Valley Research and Extension Center, Redfield, South Dakota.

Described by: Dr. C. J. Frazee, Plant Science Department, South Dakota State University.

Parent Material: Laminated Lacustrine Silt.

Horizon	Depth (cm)	Description
Ap	0-23	Very dark gray (10YR3/1) moist; silt loam; weak fine and moderate granular structure; very friable when moist; abrupt smooth boundary; noncalcareous.
B21	23-27	Dark brown (10YR3/3) moist; silt loam; weak medium prismatic structure parting to weak medium subangular blocky structure; very friable when moist; clear smooth boundary; noncalcareous.
B22	37-48	Olive brown (2.5Y5/4) moist; silt loam; weak coarse prismatic structure; very friable when moist; clear smooth boundary; noncalcareous.
C1ca	48-85	Olive brown (2.5Y5/4) moist; silt loam; massive; friable when moist; clear smooth boundary; highly calcareous.
C2	85-150	Olive brown (2.5Y4/4) moist with 10YR5/6 iron stains between plates; laminated silt loam; medium moderate plates; friable when moist; highly calcareous.

70 km northwest of the plots. Irrigation water was pumped from the James River to the farm, a distance of about 800 meters.

Field plots were established during the summer of 1976 as shown in Figure I-1. Table I-2 is a compilation of proposed treatments. The late summer and early fall weather of 1976 was a continuation of the severe drought conditions which were experienced throughout South Dakota in the summers of 1975 and 1976. In the fall of 1976, despite the stagnant flow level of the James River, we were able to pump enough water to irrigate the plots of the fall irrigated treatments with 15 cm water. Tables I-3, I-4, I-5 and I-6 are summaries of cultural practices which were applied to the corn, winter wheat, spring wheat and alfalfa plots over the time period of this study.

The corn plots were planted using a Orthman Level Bed Planter which creates a furrow system while planting the crop. A soil ridge was formed between plots to retain irrigation water on the plots. The corn was irrigated by flooding the furrows from a gated pipe at the head of each plot. Extremely wet conditions in the late summer made it impossible to irrigate at the blister kernel stage of growth.

The winter wheat plots were planted with a conventional small grain drill on land that had been summer fallowed. In the late fall and early winter months the plots had an extremely good seedling stand. Several severe rain-sleet-snow storms caused extensive winter-kill which resulted in early spring plant populations reduced more than 50%. Most local farmers experienced the same problem and plowed their winter wheat under to plant spring wheat or some other crop.

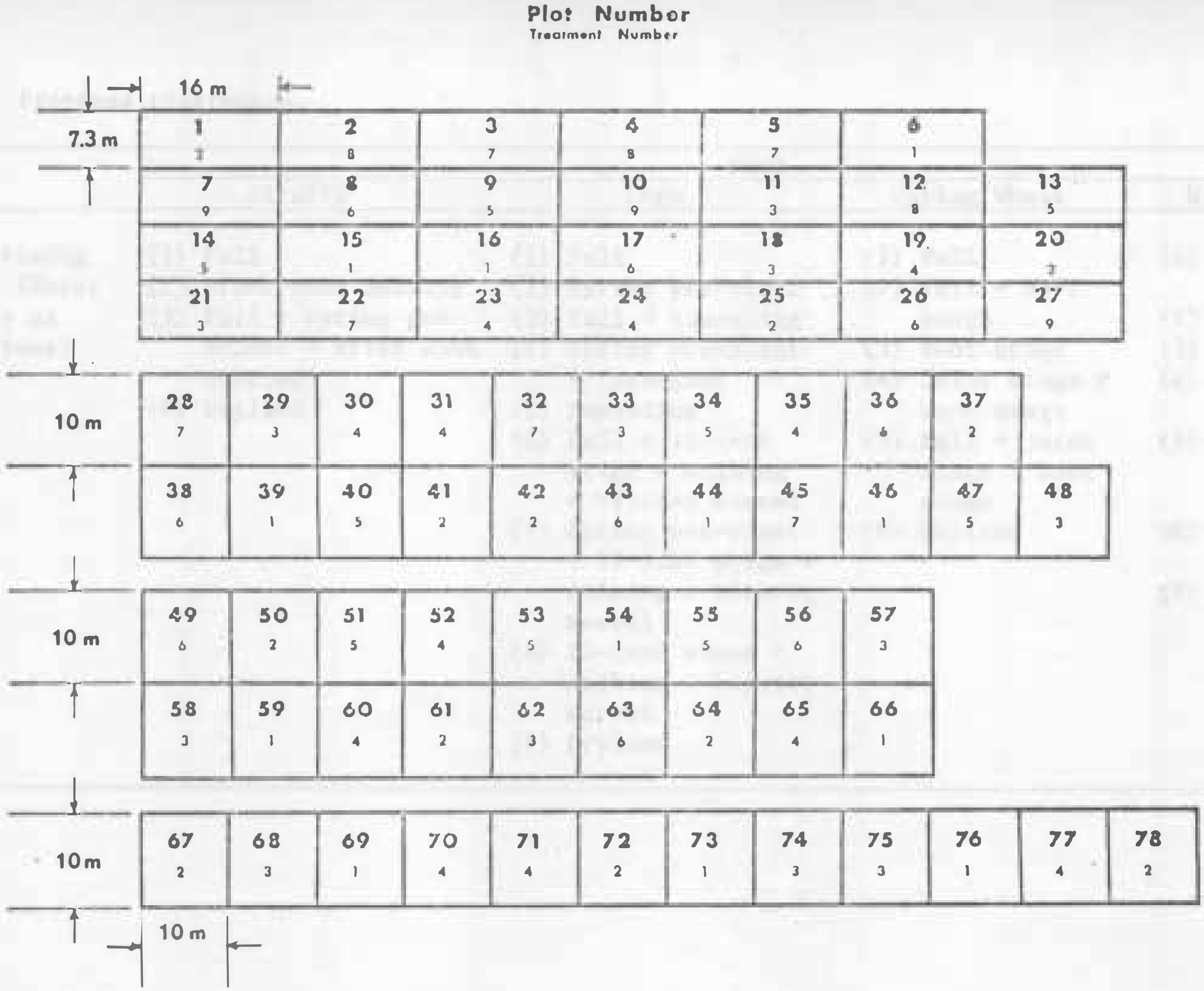


Figure I-1. Redfield plot diagram.

Table I-2. Proposed treatments.

	Crop			
	Alfalfa	Corn	Spring Wheat	Winter Wheat
Irrigation timing treatments. (Water applied only at specified times)	(1) Fall (2) After each cutting (3) Fall + spring pre-season + after each cutting (4) Dryland	(1) Fall (2) Spring pre-plant (3) Fall + tasseling (4) Spring pre-plant + tasseling (5) Tasseling (6) Fall + 12-leaf stage + silking + blister kernel (7) Spring pre-plant + 12-leaf stage + silking + blister kernel (8) 12-leaf stage + silking + blister kernel (9) Dryland	(1) Fall (2) Fall + boot stage (3) Boot stage (4) Joint stage + boot stage (5) Fall + joint stage + boot stage (6) Dryland	(1) Fall pre-plant (2) Joint stage (3) Boot stage (4) Fall + boot stage (5) Fall + joint stage + boot stage (6) Joint stage + boot stage (7) Dryland

Table I-3. Cultural practices for corn plots, 1976-77.

October 21, 1976

Treatments 1, 3, and 6 (fall irrigation) were irrigated with 15.2 cm water.

May 6, 1977

Fertilizer application (174 kg/ha N, 66 kg/ha P_2O_5, 22.4 kg/ha K_2O).

May 10, 1977

Plots were tilled using Orthman tribed splitter. Plots were then planted with the Orthman tribed planter at a population of 61280 plants/hectare. The corn variety was OsGold SX110. Herbicide used was Lasso-Bladex applied at a rate of 2.24 kg/ha active Lasso and 2.24 kg/ha active Bladex. Insecticide used was Dyfonate applied at a rate of 5.6 kg/ha active.

June 15, 1977

Cultivated all plots.

June 30, 1977

Treatments 2, 4, and 7 (12 leaf stage) were irrigated with 10.16 cm of water.

July 21, 1977

Treatments 3, 4, 5, 6, 7 and 8 (tassel and silking stages) were irrigated with 10.16 cm of water.

October 13, 1977

All plots were hand picked and yields taken.

Table I-4. Cultural practices for winter wheat, 1976-77.

September 9, 1976

Plots were disked and planted with a conventional grain drill. Variety used was Centurk planted at a rate of 67 kg/ha.

October 21, 1976

Treatments 1, 4, and 5 (fall irrigation) were irrigated with 15.24 cm of water.

May 19, 1977

Plots were treated with 0.56 kg/ha 24 Damine herbicide.

May 20, 1977

Treatments 5 and 6 (joint stage) were irrigated with 10.16 cm of water.

July 6, 1977

Swathed all plots.

July 15, 1977

Plots were harvested with combine and yields were recorded out of combine.

Table I-5. Cultural practices for spring wheat, 1976-77.

October 21, 1976

Treatments 1, 2, and 5 (fall irrigation) were irrigated with 15.24 cm of water.

October 24, 1976

Plots 51, 53, and 55 were irrigated with 10 cm of water.

April 7, 1977

Fertilizer application (56 kg/ha N, and 28 kg/ha P_2O_5).

April 25, 1977

Disked and planted using conventional grain drill. Variety used was Era seeded at a rate of 84 kg/ha.

May 19, 1977

Treatments 4 and 5 (joint stage) were irrigated with 10.16 cm of water.

July 25, 1977

Swathed all plots.

August 2, 1977

Plots were harvested with combine and yields were recorded out of combine.

Table I-6. Cultural practices for alfalfa, 1976-77.

April 14, 1976

Agate certified alfalfa was seeded at a rate of about 15 kg/ha without a cover crop.

April 14-July 19, 1976

All plots were treated alike with irrigations applied to establish uniform stands.

October 21, 1976

Treatments 1 and 3 (fall irrigation) irrigated with 15.24 cm of water.

May 3, 1977

Treatment 3 (spring preseason) irrigated with 10.2 cm of water.

May 25, 1977

Harvest all plots and irrigated treatments 2 and 3 (after each cutting) with 10.2 cm of water.

July 5 and 6, 1977

Harvested all plots and irrigated treatments 2 and 3 (after each cutting) with 12.7 cm of water.

August 10, 1977

Harvested all plots.

Because of the poor stand, moisture monitoring on the winter wheat plots was discontinued; however, irrigation treatments were continued. Heavy rains at the boot stage eliminated that planned irrigation in the winter wheat. The wheat was harvested with a combine.

The spring wheat was also planted on ground that was kept in summer fallow condition during the 1976 growing season. Plots 51, 53 and 55 were covered with black plastic in an attempt to simulate a zero flux top boundary value. These plots are considered in chapter III of this work. The spring wheat was planted using a conventional grain drill. The stand appeared to be quite uniform across all plots throughout most of the growing season. The boot stage irrigation was eliminated due to heavy precipitation.

In addition to the discussion of our field plots, the method of data analysis for the overwinter water loss portion of this study will be examined.

There are basically five criteria that must be considered when addressing the question of fall irrigation. They are: 1. Percent loss of water between fall and spring which the irrigator is willing to accept, assuming normal weather conditions. Percent loss is defined analytically as

$$PL = [1-((AI-ANI)/AFIA)]*100 \qquad [1]$$

where

PL = percent loss (%)

AI = amount of water contained within the root zone of a fall irrigated plot as measured the following spring (cm)

ANI = amount of water contained within the root zone as measured the following spring of a plot having the same surface cover as the fall irrigated plot and the same moisture content prior to the time of fall irrigation but not fall irrigated (cm)

AFIA = amount of water applied at fall irrigation (cm)

2. The second criterion is the question of what moisture the irrigator feels he needs within his soil profile at the beginning of the following spring. In determining this parameter, the management system of the irrigator coupled with the availability of energy and water in both the spring and fall are of critical importance. There would be different needs when comparing an 1800 gpm, 520 acre system with a 300 gpm, 35 acre system or a 700 gpm, 130 acre system being used on three different fields. It is probable that in the future irrigation during peak power usage times will be curtailed by allocation or price structuring or eliminated because of a lack of availability.

3 and 4. The third and fourth criteria relate to the probability of too much spring moisture. An excess of moisture can cause delays in farming operations, especially in finer textured soil profiles. In addition, wet soil profiles warm up more slowly in the spring than do dry profiles.

5. The last criteria is the question of increased leaching as a result of fall irrigation. In the Great Plains, the need for salinity leaching is most significant. Since salinity problems are not within the scope of this thesis, no further discussion will be made regarding the leaching potential of fall applied irrigation water.

The data used for the overwinter analysis will be analyzed for the time period October 26, 1976 (Julian date 6299) through and including April 7, 1977 (Julian date 7097). The above dates were chosen so that for the four different crops and cultural practices under study the dates would be late enough and early enough, respectively, to avoid tillage practices and water loss resulting from transpiration. In the interest of comparing data between crops, computations were evaluated only to a depth of 120 cm even though data were collected to greater depths in some of the plots.

During the period of study, a total of 22.78 cm of precipitation was recorded at the Redfield farm weather station of which 16.77 cm were rain and 6.15 cm came as snow. The historical average (1897 to 1969) for the study period was 9.12 cm precipitation with about 5.85 cm of the total falling as snow (1897-1969 U.S. Department of Commerce, Climatological Summary). The precipitation during the study period was just about double the average normal precipitation but the precipitation as snowfall was quite close to normal.

Table I-7 is a summary of the moisture release characteristics of the Great Bend silt loam soil on which these experiments were conducted (Stone 1973) and (Frankenstein 1973). The data shown in Table I-7 are useful for interpreting moisture characteristics of the plots.

In September of 1976, South Dakota was in the most severe drought cycle since the 1930's (U.S. Department of Commerce, Climatological Summary) and both surface soil and subsoil moisture conditions were extremely dry. If the practice of fall irrigation were ever to be

Table I-7. Moisture release data for Great Bend silt loam soil.

Soil water pressure (cm of water)	Depth (cm)								
	0	15	30	50	70	90	110	130	150
	Soil water content (cm^3/cm^3)								
- 5	0.433	0.443	0.442	0.436	0.468	0.497	0.521	0.515	0.525
- 20	0.381	0.393	0.410	0.412	0.456	0.490	0.507	0.506	0.516
- 40	0.354	0.366	0.386	0.395	0.445	0.483	0.499	0.500	0.511
- 60	0.340	0.352	0.371	0.384	0.438	0.477	0.494	0.496	0.507
- 90	0.328	0.340	0.355	0.372	0.429	0.470	0.488	0.492	0.503
- 130	0.318	0.330	0.341	0.361	0.419	0.462	0.483	0.488	0.499
- 180	0.311	0.323	0.330	0.350	0.409	0.455	0.478	0.485	0.495
- 240	0.304	0.316	0.319	0.339	0.399	0.447	0.474	0.481	0.492
- 310	0.298	0.309	0.311	0.330	0.390	0.439	0.469	0.478	0.488
- 400	0.292	0.302	0.302	0.319	0.379	0.431	0.463	0.473	0.483
- 800	0.273	0.286	0.237	0.292	0.346	0.414	0.452	0.465	0.475
-15200	0.202	0.202	0.182	0.189	0.189	0.174	0.192	0.192	0.203
	Soil bulk density (g/cm^3)								
	1.15	1.17	1.19	1.16	1.24	1.22	1.18	1.23	1.23

economically feasible, the drought conditions of 1976 were certainly favorable to show this result. Fortunately for the agriculture of the area but less fortunately for our irrigation experiment, the early spring of 1977 was an extremely wet period for South Dakota and these weather conditions negated the possibility of showing statistically significant yield gain by fall irrigation.

If we could devise an analytical procedure for predicting the probable PL given an average production year, we would be in a better position to analytically estimate how much fall irrigation water should be applied. To develop PL function, the component variables must be considered. An initial attempt at developing a function follows:

PL = f(Rain, Effective snow, Time, Potential evapotranspiration Moisture content after irrigation of profile, Initial moisture content, Runoff, Total water holding capacity of the soil profile) [2]

Because there is very little relevant theory that is usable to further develop this functional relationship, the only available means of approximation is a statistical regression approach. In a first analysis of each of the above terms, there appeared to be no great interdependence of terms and, thus, an additive linear model appeared feasible.

To generate a statistical equation of the form (PL = A+B Rain + C Snow...) data from many different locations taken over several years should be used. At our disposal were data taken from four different crop covers on the same soil over one winter. The time period from 6299-7097 was divided up into five time intervals each starting and ending on days when soil moisture data were available. Table I-8 is

Table I-8. Weather summary.

Date	Rain (cm)	Snow (cm)	Time (days)	ETp (cm/time period)
Oct. 26, 1976 (6299)- Nov. 9, 1976 (6313)	2.95	0	14	2.49
Nov. 9, 1976 (6313)- Nov. 19, 1976 (6323)	0	0	10	1.59
Nov. 19, 1976 (6323)- Dec. 15, 1976 (6349)	0	1.96	26	1.39
Dec. 15, 1976 (6349)- Jan. 21, 1977 (7021)	0	.72	37	.79
Jan. 21, 1977 (7021)- April 7, 1977 (7097)	13.82	3.33	56	6.53
Sub-total	16.77	6.01	143	12.79

Total precipitation = 22.78

a summary of the weather records and time intervals used. For regression analysis with eight independent variables it is necessary to have at least nine sets of data. To meet this requirement five data sets were used from the five time intervals. By combining the simultaneous pairs of data from the initial five data sets, four more data sets became available. Three sets of data were obtained by combining every combination of three simultaneous time intervals. Likewise two sets were available from four time intervals and one set came from the total time frame. This results in 15 sets of data or equations for each of the four crop covers. The average of the dryland plots was the assumed moisture content before irrigation. The assumption (not totally valid) was made that an irrigation was made at the beginning of each time period bringing the profile moisture content up to the level of the actual measured irrigated plot moisture content.

Eight independent variables are listed in the functional relationship described in equation [2]. Rainfall and snowfall are the first and second variables listed in equation [2]. As we consider function [2] it seems clear that the moisture input parameters, snow and rain, should be of considerable importance in the functional relationship. Even though these two factors vary greatly from one year to the next, using historical data we can predict with a known probability the expected rainfall and snowfall for any year. When the amount of rain and snow which infiltrates exceeds field capacity minus ANI (ANI taken from equation [1]), then PL must be 100% and any additional water would go to runoff or drainage. If, however, there is no rain or snow infiltrating, PL will probably be close to zero (not equal to zero) assuming that AFIA does not bring the profile to a moisture content which exceeds field capacity.

In equation [2] the term effective snowfall is a function not only of the total precipitation that falls as snow but also includes a surface roughness factor which attempts to analytically describe the ability of a particular surface to hold snow. For a particular field, a quantitative value describing roughness would be dependent upon the depth and density of crop residue. As an example, we would speculate that an alfalfa stubble 10 cm in height would be more dense than a corn stubble 10 cm in height and would have a greater ability to trap snow, thus, we should assign it a higher roughness factor than the corn stubble. In an attempt at developing some type of objective function, we analyzed data from nonirrigated plots during a time when precipitation fell only as snow. The reason for using nonirrigated plots was that we could make the assumption (not totally valid) that all gain in moisture in the profile was a result of effective snowfall. To develop an equation we made the assumption that

$$ESF = ASF\ (A+BR+CR^2) \qquad [3]$$

where

ESF = effective snowfall (cm as liquid water)

ASF = actual snowfall (cm as liquid water)

R = surface roughness factor (dimensionless)

A, B, and C = regression constants

Our alfalfa stood about 30 cm tall which we felt would be a maximum cover so we gave the alfalfa an arbitrary value of R equal to 10. The corn stubble averaged about 18 cm tall but was considerably less dense so we assigned to it a value of R equal to 3. The winter wheat had a cover about 5 cm tall. The spring wheat, although fallow, had some trash and its roughness was similar to that of the winter wheat. We assigned a value of R equal to 1 to these plots. The ASF for the period of October 19 to January 21 was 2.67 cm (liquid water). Table I-9 is a compilation of the data used. The equation developed from the data in Table I-9 is

$$\frac{ESF}{ASF} = 0.73+.034R+.022R^2 \qquad [4]$$

with

$r^2 = 0.72$

$S = 0.68$

Although the r^2 value is reasonable, additional data are needed to determine the validity of the equation because it tells us that when R is equal to zero then ESF/ASF will be 0.73 or 73%.

The next term to be evaluated is the time function. Our objective for putting the time function into the equation was that it might act as a drainage term. Looking at the correlation analysis of the composite of all crops Table I-26 we find that the time function correlates highly (above .75) with rain, snow and ETp. This result should have been foreseen since rain, snow and ETp are clearly time dependent. Because of the high correlation with these supposedly independent variables it would be advisable to exclude this term from the functional

Table I-9. Analysis of effective snowfall (ESF).

Alfalfa plots				Corn plots				Winter and spring wheat plots			
Plot	cm/120 cm Nov. 19 (6323)	cm/120 cm Jan. 21 (7021)	Difference	Plot	cm/120 cm Nov. 19 (6323)	cm/120 cm Jan. 21 (7021)	Difference	Plot	cm/120 cm Nov. 19 (6323)	cm/120 cm Jan. 21 (7021)	Difference
67	27.9	35.8	7.9	9	38.1	42.1	4.0	29	47.6	49.2	1.6
72	26.7	32.0	5.3	13	33.9	35.0	1.1	33	48.9	51.4	2.5
78	31.2	39.3	8.1	14	41.1	44.3	3.2	48	48.4	50.5	2.1
70	28.9	42.7	13.8	7	39.7	43.9	4.2	28	44.4	47.1	2.7
71	28.1	38.5	10.4	10	36.2	40.2	4.0	32	48.6	51.8	3.2
77	31.2	37.9	6.7	27	37.4	37.4	0.0	45	45.1	45.1	0.0
		Mean	8.70			Mean	2.75				
		S	3.02			S	1.77	49	47.2	50.2	3.0
								56	49.3	51.8	2.5
								63	49.4	50.7	1.3
										Mean	2.10
										S	.87

relationship. Regression analysis also bears this out.

The moisture content before irrigation and moisture content after irrigation are included to provide an estimate of position on the moisture release curve at a particular time.

When utilizing this type of analysis in the broader context of a general equation for different soils, it would be better to develop the rain, snow and ETp terms in the context of percent of the moisture between saturation and field capacity and the moisture before irrigation and moisture after irrigation as a percent of saturation.

The runoff term is important; however, in our study, the dikes around the plots eliminated any runoff.

The total soil moisture holding capacity term should be a significant variable but with data taken from only one location the possibility of inclusion of this term was non-existent.

SECTION 3

RESULTS AND DISCUSSION OF OVERWINTER WATER CONSERVATION

Tables I-10, I-11, ..., I-15 are summaries of the average soil moisture content by treatment on selected sampling days. Tables I-16 and I-17 are summarizations of the net change in soil moisture content over the study period. Tables I-18, I-19, ..., I-27 are tables of correlation coefficients and stepwise linear regression coefficients for the PL function (PL = A+B Rain + C Snow + ...) for each crop and the composite of all crops. Each crop will be discussed briefly.

Starting with alfalfa and looking at Table I-10 the moisture content of the non-fall irrigated plots (treatments 2 and 4) was 29.25 cm/120 cm which is 24.4% by volume. This value is a little greater than halfway between field capacity and permanent wilting point as seen in Table I-7. The fall irrigated plots (treatments 1 and 3) averaged 48.70 cm/120 cm which is 40.6% by volume. Although the area had been in a state of severe drought, the dryland plots did not reflect the dryness of some of the surrounding alfalfa fields because on several occasions during the summer of 1976 all plots were irrigated to insure the establishment of a uniform alfalfa stand. During the first period of time (Julian date 6299-6313) even though 5 cm of rain (Table I-7) fell, both the irrigated and nonirrigated plots show a net loss of moisture (Table I-16). This indicates that the plants were still evapotranspiring. This is further substantiated by the nonirrigated spring wheat and the corn (Table I-16) both of which showed a net positive gain. As soon as precipitation began falling as snow the alfalfa

Table I-10.

Moisture contents on 6299, replication means (cm water/120 cm depth)

Treatment #	Crop: Alfalfa	Spring wheat	Winter wheat	Corn
1	49.7**	55.6**	56.1**	53.6**
2	28.7		49.3*	34.1
3	47.7**		48.5	51.9**
4	29.8		57.4**	37.1
5		55.0**	55.8**	37.0
6		47.5	47.4	53.1**
7			46.8	39.3
8				38.2
9				36.7
Mean	38.98	52.7	51.61	42.33
S	11.27	4.51	4.60	8.03
F	138.9**	99.2**	24.4**	28.7**

Table I-11.

Moisture content on 6313, replication means (cm water/120 cm depth)

Treatment #	Crop: Alfalfa	Spring wheat	Winter wheat	Corn
1	48.7**	53.3**	52.4**	50.7**
2	28.6		49.5*	35.1
3	47.5**		48.3	50.0**
4	29.4		53.1**	39.4
5		54.4**	52.4**	37.7
6		48.6	46.4	50.9**
7			46.1	39.7
8				38.5
9				37.8
Mean	38.55	52.1	49.7	42.2
S	11.04	3.08	2.94	6.39
F	97.9**	15.3**	9.4**	22.9**

Table I-12.

Moisture content on 6323 replication means (cm water/120 cm depth)

Treatment #	Crop: Alfalfa	Spring wheat	Winter wheat	Corn
1	48.0**	52.1**	51.6**	50.2**
2	28.6		49.1*	35.7
3	46.6**		48.3	49.4**
4	29.2		52.2**	40.4
5		52.8**	51.6**	37.8
6		48.2	46.9	50.1**
7			46.2	40.0
8				39.0
9				37.9
Mean	38.10	51.03	49.41	42.28
S	10.64	2.48	2.43	5.88
F	84.0**	8.8*	8.1**	17.7**

Table I-13.

Moisture content on 6349, replication means (cm water/120 cm depth)

Treatment #	Crop: Alfalfa	Spring wheat	Winter wheat	Corn
1	48.3**	52.5*	51.4**	49.5**
2	29.6		49.9*	35.8
3	46.6**		49.0*	48.2**
4	30.6		52.0**	41.2
5		53.5*	51.1**	37.3
6		49.3	47.6	48.2**
7			46.4	40.1
8				37.3
9				37.6
Mean	38.78	51.77	49.63	41.69
S	10.05	2.19	2.08	5.46
F	59.1**	4.095	4.51**	8.17**

Table I-14.

Moisture content on 7021, replication means (cm water/120 cm depth)

Treatment #	Crop Alfalfa	Spring wheat	Winter wheat	Corn
1	53.3**	53.2*	52.7**	51.4**
2	35.7		51.8*	39.0
3	51.5**		50.4	51.3**
4	39.7		53.6**	43.9
5		55.0**	52.6**	40.5
6		50.9	48.9	53.1**
7			48.0	43.2
8				42.1
9				40.5
Mean	45.05	53.03	51.14	45.0
S	8.67	2.06	2.10	5.43
F	31.24**	7.65*	3.03*	8.70**

Table I-15.

Moisture content on 7097, replication means (cm water/120 cm depth)

Treatment #	Crop Alfalfa	Spring wheat	Winter wheat	Corn
1	56.9**	58.9*	58.7*	55.8
2	49.1		58.3	52.0
3	55.4**		57.9	55.1
4	51.4		58.9*	54.6
5		58.4	58.6*	52.9
6		57.4	56.6	56.1
7			56.5	53.1
8				51.5
9				52.3
Mean	53.20	58.23	57.93	53.71
S	3.59	.76	.99	1.72
F	28.21**	3.8	2.51	1.67

Table I-16.

Net gain and loss of all plots by crop (cm water/120 cm depth)

Date	Alfalfa	Spring wheat	Winter wheat	Corn
6299-6313	-.43	- .60	-1.91	-.13
6313-6323	-.45	-1.07	- .29	.08
6323-6349	.68	.74	.22	-.59
6349-7021	6.27	1.26	1.51	3.31
7021-7097	8.15	5.20	6.79	8.71
Net	14.22	5.53	6.32	11.38

Net gain and loss of fall irrigated plots by crop (cm water/120 cm depth)

Date	Alfalfa	Spring wheat	Winter wheat	Corn
6299-6313	-.60	-1.45	-3.80	-2.34
6313-6323	-.80	-1.40	- .83	- .63
6323-6349	.15	.55	- .30	-1.27
6349-7021	4.95	1.10	1.47	3.30
7021-7097	3.75	4.55	5.76	3.74
Net	7.45	3.35	2.30	2.74

Net gain and loss of non-fall irrigated plots by crop (cm water/120 cm depth)

Date	Alfalfa	Spring wheat	Winter wheat	Corn
6299-6313	-.25	1.10	- .42	.96
6313-6323	-.10	- .40	.05	.44
6323-6349	1.20	1.10	.60	- .25
6349-7021	7.60	1.60	1.55	3.31
7021-7097	12.55	6.50	7.55	11.20
Net	21.00	9.90	9.33	15.66

Table I-17.

Change in moisture content from 6299 to 7097, replication means (cm water/120 cm depth)

Treatment #	Crop: Alfalfa	Spring wheat	Winter wheat	Corn
1	7.2	3.3	2.6	2.2
2	20.4		9.0	17.9
3	7.7		9.4	3.2
4	21.6		1.5	17.5
5		3.4	2.8	15.9
6		9.9	9.2	3.0
7			9.7	13.8
8				13.3
9				15.6
Mean	14.2	5.5	6.3	11.4
S	7.8	3.8	3.8	6.6

Table I-18.

Correlation Coefficients for Alfalfa Overwinter Soil Profile

	Rain (cm)	Snow (cm)	Time (days)	ETp (cm)	Mc before (cm/120 cm)	Mc after (cm/120 cm)
Snow	.81					
Time	.77	.95				
ETp	.90	.86	.90			
Mc before	.36	.06	-.05	.11		
Mc after	.26	-.01	-.06	.18	.72	
% loss	.94	.91	.88	.86	.32	.13

Table I-19.

Step-wise Linear Regression, PL = f (Rain, Snow ... etc.)

Constant	Rain	Snow	ETp	Time	Mc before	Mc after
.082	.039					
SE = .094 = 9.4%		MCC = .940		F = 211**		
.012	.024	.057				
SE = .064	MCC = .974		F = 34.0**			
.048	.031	.072	-.022			
SE = .058	MCC = .979		F = 7.33**			
.039	.039	.017	-.047	.004		
SE = .044	MCC = .988		F = 1.31			
-.406	.033	.013	-.043	.004	.014	
SE = .037	MCC = .992		F = 1.04			
.052	.031	.012	-.035	.004	.023	-.015
SE = .035	MCC = .993		F = .936			

Table I-20.

Correlation Coefficients for Spring Wheat Overwinter Soil Profile

	Rain (cm)	Snow (cm)	Time (days)	ETp (cm)	Mc before (cm/120 cm)	Mc after (cm/120 cm)
Snow	.81					
Time	.77	.95				
ETp	.90	.86	.90			
Mc before	.25	.11	-.04	-.07		
Mc after	-.01	-.21	-.06	.21	-.43	
% loss	.79	.73	.78	.88	-.07	.27

Table I-21.

Step-wise Linear Regression, PL = f (Rain, Snow ... etc.)

Constant	ETp	Mc after	Rain	Time	Snow	Mc before
-.136	.062					
SE = .118 = 11.8%		MCC = .88	F = 97**			
-.838	.061	.018				
SE = .118	MCC = .885	F = 1.01				
-1.030	.055	.022	.004			
SE = .120	MCC = .886	F = .164				
-1.964	.030	.039	.008	.001		
SE = .121	MCC = .888	F = .532				
-1.699	.032	.034	.009	.002	-.013	
SE = .124	MCC = .889	F = .065				
-1.633	.031	.035	.009	.002	-.012	-.002
SE = .126	MCC = .889	F = .002				

Table I-22.

Correlation Coefficients for Winter Wheat for Overwinter Soil Profile

	Rain (cm)	Snow (cm)	Time (days)	ETp (cm)	Mc before (cm/120 cm)	Mc after (cm/120 cm)
Snow	.81					
Time	.77	.95				
ETp	.90	.86	.90			
Mc before	.38	.00	-.05	.14		
Mc after	.02	-.18	-.02	.24	.09	
% loss	.78	.71	.75	.92	.15	.45

Table I-23.

Step-wise Linear Regression, PL = f (Rain, Snow ... etc.)

Constant	ETp	Mc after	Time	Snow	Rain	Mc before
.153	.060					
SE = .087 = 8.7%		MCC = .924	F = 249**			
-1.175	.056	.026				
SE = .070	MCC = .952	F = 24.2**				
-.899	.068	.020	-.001			
SE = .069	MCC = .955	F = 2.10				
-1.481	.057	.031	-.002	.042		
SE = .067	MCC = .958	F = 3.50*				
-1.301	.074	.025	-.003	.042	-.006	
SE = .066	MCC = .960	F = 1.30				
-1.559	.076	.024	-.003	.042	-.007	.010
SE = .067	MCC = .960	F = .134				

Table I-24.

Correlation Coefficients for Corn Overwinter Soil Profile

	Rain (cm)	Snow (cm)	Time (days)	ETp (cm)	Mc before (cm/120 cm)	Mc after (cm/120 cm)
Snow	.81					
Time	.77	.95				
ETp	.90	.86	.90			
Mc before	.27	.18	-.03	-.04		
Mc after	.06	-.12	-.03	.25	-.31	
% loss	.97	.82	.79	.92	.21	.15

Table I-25.

Step-wise Linear Regression, PL = f (Rain, Snow ... etc.)

Constant	Rain	ETp	Mc before	Time	Snow	Mc after
.122	.044					
SE = .074 = 7.4%		MCC = .969	F = 650**			
.065	.033	.023				
SE = .066	MCC = .976		F = 12.6**			
-.584	.028	.031	.017			
SE = .065	MCC = .977		F = 1.83			
-.720	.027	.043	.020	-.001		
SE = .064	MCC = .978		F = 2.28			
-.460	.026	.043	.014	-.002	.021	
SE = .064	MCC = .979		F = .75			
-.562	.027	.039	.013	-.002	.020	.003
SE = .065	MCC = .979		F = .072			

Table I-26.

Correlation Coefficients for All Crops for Overwinter Soil Profile

	Rain (cm)	Snow (cm)	Time (days)	ETp (cm)	Mc before (cm/120 cm)	Mc after cm/120 cm)
Snow	.81					
Time	.77	.95				
ETp	.90	.86	.90			
Mc before	.05	.01	-.01	.01		
Mc after	.04	-.08	-.02	.14	.78	
% loss	.85	.76	.77	.87	.26	.34

Table I-27.

Step-wise Linear Regression, PL = f (Rain, Snow ... etc.)

Constant	ETp	Mc before	Rain	Mc after	Snow	Time
.054	.067					
SE = .130	MCC = .873	F = 472**				
-.324	.067	.009				
SE = .111	MCC = .909	F = 55.6**				
-.270	.047	.009	.012			
SE = .106	MCC = .918	F = 15.02**				
-.851	.039	.004	.016	.015		
SE = .104	MCC = .922	F = 4.33**				
-1.423	.018	-.0001	.018	.030	.031	
SE = .102	MCC = .926	F = .002				
-1.448	.015	-.0003	.019	.031	.027	.0003
SE = .102	MCC =	F = .012				

plots with a considerable cover from early fall growth became effective snow trappers. This was especially true from 6349 to 7021 when only 0.72 cm of actual precipitation fell while all alfalfa plots recorded a net gain of 6.27 cm (Table I-16) due to trapped snow.

From the data of Table I-16 it is quite clear that the alfalfa with its dense cover, trapped considerably more snow than did any of the other plots. The dryland alfalfa plots ended up gaining a total of 21 cm of moisture (alfalfa plots 2 and 4 of Table I-17). The alfalfa was the only crop in the irrigated treatments that was statistically significantly different from the dryland treatments (Table I-15) on Julian date 7097.

The stepwise regression (Table I-19) analysis of the PL loss function tells us that the three significant (greater than .05) variables in the alfalfa are rain, snow and ETp. The ability of alfalfa to trap snow makes it clearly superior to either corn or wheat cover in retaining moisture which comes in the form of snow.

Spring wheat and winter wheat are considered as a composite since they closely resemble each other in the data analysis. Both crops started out in the fall in a wet condition (Table I-10) after summer fallow. The spring wheat dryland treatments averaged 47.5 cm/120 cm which is 39.58% by volume and the winter wheat dryland treatments averaged 48.0 cm/120 cm which is 40% by volume. The fall irrigated plots were 55.3 cm/120 cm which is 44.42% by volume, and 56.43 cm/120 cm, which is 47.03% by volume, so both plots exceeded the 1/3 bar moisture percentage. Over time, Tables I-11 through I-15 show a rapid and real decay of the differences between moisture content of the irrigated and nonirrigated plots. On Julian date 7097 there is only 1.25 cm difference in water content between the irrigated and nonirrigated spring wheat and 1.40 cm of difference in the winter wheat. These differences were barely statistically significant. For practical crop production purposes, they were not significantly different.

In the regression analysis, the only significant variable in the spring wheat (Table I-21) was ETp. In the winter wheat stepwise regression (Table I-23), the ETp, MC after, and snow variables were significant. The MC after is the term indicating how much water was applied and it should be significant; however, the snow term for winter wheat might not be expected to be significant.

The corn crop in almost every way was a compromise between the alfalfa and the wheat. From a surface-roughness or cover standpoint, it was in between the wheat and alfalfa. Corn plots had considerably more cover than was found on the wheat but considerably less cover than the alfalfa. The corn treatment had greater initial moisture contents (Table I-10) than did the alfalfa with 47.5 cm/120 cm which is 39.6% by volume, and 55.3 cm/120 cm which is 46.1% by volume in the irrigated plots. The corn plots collected snow better than did the wheat plots but not as well as the alfalfa plots (Table I-17). At the last observation period, there was no statistical difference between the moisture content of irrigated and nonirrigated corn plots.

The stepwise regression found the variables rain and ETp to be significant in that order.

Looking at the composite regression equation (Table I-27), ETp is found to be the most significant variable, moisture content before irrigation is the second most significant variable, rainfall is the third most significant variable and moisture content after irrigation is the least significant variable. In theory, if our assumption of the type of function is correct, and if the methodology of correcting for surface roughness is correct, then there should be little or no differences between the equations generated from one crop to another. In other words, the coefficient for the independent crops (Tables I-19, I-21, I-23, and I-25) should be equal to the coefficient of the composite variable. Even though our composite equation accounts for greater than 90% of the variation, it is clear that, since all criteria are not met, more work is needed in this area to obtain a reliable method of predicting percent moisture loss.

If we can assume that a valid composite equation of this form can be found and if we assume an average amount of precipitation with average evaporating conditions, we can look at an actual fall soil moisture profile and predict, given a PL we will willingly accept, how much water we should apply.

Assumptions:

1. The composite equation using all variables in Table I-27 is a valid and useful equation.
2. Over winter rainfall will be 3.27 cm.
3. Over winter snowfall will be 5.85 cm.
4. Potential evapotranspiration for the period will be 14 cm.

5. The total time period covered will be 180 days.

6. The initial moisture contents will be 20 cm/120 cm, 30 cm/ 120 cm and 40 cm/120 cm.

7. We are willing to accept PL of 10%, 30%, and 50%.

Table I-28 is an example of an analysis to determine how much water should be applied as fall irrigation given specific constraints. As we can see, the amount varies from none to 27 cm depending upon how much loss we are willing to accept and how much water was initially in the profile.

Table I-28. Example of analysis to determine fall irrigation requirements from percent loss function.

Acceptable PL	Initial moisture content, cm/120 cm 20	30	40
	cm water to apply as fall irrigation		
10	14.51 cm	4.61	----
30	20.97 cm	11.06	1.16
50	27.42 cm	17.51	7.61

CHAPTER II

SECTION 1

INTRODUCTION AND LITERATURE REVIEW OF CROP MODEL

If the results obtained from a predictive equation as established in the past chapter are to be of any value, it is necessary to the development of some production function which can convert the amount of moisture available in the spring into crop yield.

The development of crop production functions with the capabilities of estimating the response to variable quantities of water, made available at different times during the growing season is necessary to further our management ability.

There are two basic schools of thought which must be considered concerning the availability of water to plants. The first theory (Veihmeyer and Hendrickson, 1955) maintains that the utilization of soil moisture by plants is uniformly effective throughout the entire range between permanent wilting point and field capacity. The second theory supported by many scientists today (Hagen et al., 1957, Bahrani and Taylor, 1961 and Lemon et al., 1957) is that plants respond to what we can refer to as a mean soil moisture stress. This term is a combination of the soil moisture tension throughout the root zone and the evapotranspiration potential of the atmosphere.

Even though the literature has shown a real need for the development of time-dependent crop production functions (Trava et al., 1977 and Dudley, 1971) as a tool necessary for the optimization of crop production and water efficiency, there has been surprisingly little work done on the subject.

Moore (1961) breaks the growing season up into cycles and assumes that maximum growth occurs at field capacity within each cycle. Each cycle will start at an irrigation or rain and end at the next irrigation or rain. Moore assumes: 1. There is a definite relationship between plant growth and the mean moisture stress. This relationship can be determined utilizing the inverted soil moisture release curve. 2. The relative potential growth of the plant part to be harvested, be it grain or forage, is linear over time. 3. The expected yield of the crop is obtainable by summing the growth increments over the independent cycles. In mathematical terms the model looks like this.

$$G_i = \frac{\int_0^{\Theta_i} g(x)\,dx}{\Theta_i * 100} \qquad [5]$$

where

G_i = relative growth during the ith cycle as a % of potential growth (% of potential).

Θ_i = is the moisture depletion percent at which the particular ith cycle is terminated (% of depletion).

$g(x)$ = the functional relationship of relative growth to % moisture depletion found by inverting the moisture release curve (% depletion).

and for the entire season

$$Y = \sum_{i=1}^{n} G_i \frac{t_i}{T} \qquad [6]$$

where

Y = total yield (% of potential)

t_i = number of days in ith cycle

T = total number of days in growing season

Some of the shortcomings of Moore's model are: 1. The possibility that growth is not a linear function of time. 2. The model does not consider any cycle as dependent upon what occurred during the previous time periods. 3. The growth curve within each cycle is always a monotonously decreasing function.

Flinn and Musgrave (1967) divided the growing cycle up into eight 30-day periods. Within each cycle they develop a growth index function of sigmoidal shape with a different curve for each cycle. The index evaluates the number of days during which actual evapotranspiration does not exceed potential. The index term is multiplied by the potential growth for this particular cycle. After this has been accomplished the values from each cycle are added to give a seasonal expected total. Analytically the model looks like this.

$$I_i = \frac{EA_i}{D_i} \qquad [7]$$

where

I_i = index # for cycle i (dimensionless)

EA_i = number of days during cycle i during which ET actual does not exceed ET potential (days)

D_i = number of days in cycle (days)

and

$$Y = \sum_{i=1}^{n} I_i PY_i \qquad [8]$$

where

Y = total yield (bushels, tons, pounds, etc.)

PY_i = potential yield attributable to this ith portion of the season (bushels, tons, pounds, etc.)

The Flinn and Musgrave equation is another additive model which does not consider the possibility of interaction between time periods and which gives no indication as to how the potential growth for each time period was determined.

Hall and Butcher (1968) break the plant growth season into n cycles not necessarily equal and dependent more on physiological maturity rather than a definite chronological period. They postulate that if the moisture content of the soil root zone profile is maintained during every cycle at field capacity the yield will be maximized. If the soil moisture falls below field capacity then some reduction will occur. Unlike the previous additive relationship they suggest that the functional relationship should be multiplicative in nature. Analytically the Hall and Butcher model is

$$Y = a_1(w_1)*a_2(w_2)*\ldots\ a_n(w_n) \qquad [9]$$

where

Y = yield expressed as a % of max or potential yield (% of max)

$a_i(w_i)$ = a functional relation of the % of maximum yield during the ith cycle clearly dependent upon w_i which is the soil moisture content for the ith period.

Although this procedure has problems such as some independence between cycles, it appears to be an improved version. Using this formula, if the moisture content during one cycle forces the crop to death ($a_i(w_i) = 0$) then the yield will clearly be zero. Additive models will not result in this conclusion. Several problems remain that must be clarified and improved, especially with respect to the development of $a_i(w_i)$ function, before this kind of equation can be considered workable.

Hanks (1974) suggests that for planning purposes dry matter yields may be predicted accurately using the very simple function of deWit (1958) which states

$$Y = mT/E_o \qquad [10]$$

where

Y = yield (kg/ha)

m = a crop factor (kg/ha day)

T = transpiration (cm)

E_o = average free water evaporation (cm/day)

and for a given crop within a given year

$$Y/Yp = T/Tp \qquad [11]$$

where

Yp = yield potential

and Tp = transpiration potential

Hanks further suggests that the same procedure can be utilized for grain yields by dividing the growing season up into cycles or stages of growth resulting in the following equation.

$$Y(\text{grain})/Yp(\text{grain}) = (T_1/Tp_1)^{\lambda_1}\,(T_2/Tp_2)^{\lambda_2}\ldots(T_5/Tp_5)^{\lambda_5} \qquad [12]$$

where

λ_i = a weighting factor for the ith cycle or stage of growth.

Equation [12] is algebraically identical to equation [9] proposed and suggested by Hall and Butcher with the term on the right side of the equation being defined by Hanks as an equation rather than a functional relationship.

Minhas et al. (1974) presented a time dependent cyclic procedure which is very similar to Hanks (1974) and Hall and Butcher (1968); however, Minhas et al. present considerable more detail of technique. Their basic equation is written

$$Y = a\{1-(1-x_1)^2\}^{b_1}\{1-(1-x_2)^2\}^{b_2}\ldots\{1-(1-x_n)^2\}^{b_n} \qquad [13]$$

where

Y = yield (q/ha)

x_i = relative evapotranspiration during the ith period (dimensionless)

a, b_i = constants determined statistically

As with Hall, the length of period and the number of periods is variable. By squaring the $1-x_i$ term they felt that more sensitivity is given to the periods of low ET and less response is obtained at the higher ET portion of the function.

In the same paper, Minhas et al. develop an interesting approach to a functional relationship between evapotranspiration, potential evapotranspiration, and soil moisture content. They contend that

$$\frac{ET}{ET_p} = f(x) \qquad [14]$$

where

$$x = \frac{SMC-PWP}{FC-PWP}$$

ET = actual evapotranspiration (cal/cm^2 day)

ET_p = potential evapotranspiration (cal/cm^2 day)

SMC = actual soil moisture content (cm)

PWP = permanent wilting point (cm)

FC = field capacity (cm)

and

$$f(x) = (1-e^{-rx})/(1-2e^{-r\bar{x}}+e^{-rx})$$

where

$\bar{x}$ = FC-PWP = available moisture content (cm)

where

r = a constant

This procedure has one major shortcoming. It is probable that ($ET/ET_p = f(x)$) is not a unique function but varies with ET_p. If from year to year over periods of time ET_p could be considered constant, then this procedure should be acceptable. This is not the case when comparing different geographical locations.

Fogel et al. (1976) present a schematic and describe plant moisture stress as an integral over time. They define stress as the deviation of actual evapotranspiration from the potential evapotranspiration curve. Weighting factors are added to account for different effects at different stages of growth. Limited description is given as to how this concept could be applied to a seasonal crop production function.

Considerable literature has been written concerning response of several crops at different locations throughout the country to applications of irrigation and rainfall at different times during the growing season. A review is given here of some of the more important papers dealing with corn production.

Robins and Domingo (1953), working on a fine sandy loam soil, reported that moisture depletion to the permanent wilting point for a period of one or two days at tasseling or pollination reduced yields as much as 22%. If this lack of moisture was sustained for a period of six or eight days at these same stages of growth, a yield reduction of about 50% was experienced.

Denmead and Shaw (1960) reported on a study from Iowa where 20-l crocks filled with loam soil were used to grow corn. They found a reduction in grain yield resulting from moisture stress in the vegetative, silking, and ear stages to be 25%, 50%, and 21% respectively.

Holt and Timmons (1968) working in western Minnesota and eastern South Dakota on a four year study analyzed yield as a function of four variables. The variables were available soil moisture content at the time when the corn crop was 30 cm tall (SW), plant population (D), amount of precipitation from the date of the 30 cm height until 3 weeks later (P1) and precipitation from 3 weeks to 6 weeks after the 30 cm stage (P2). They concluded that for highest yields the most important stage to receive precipitation was the P2 stage. They also found that the interaction term of the SW x P2 was significant in the development of the yield function.

In an early attempt at development of a production function, Hendricks and Scholl (1943) expressed yield as a function of temperature and precipitation using an equation of the form

$$z = A_o + a_o \Sigma x + a_1 \Sigma tx + a_2 \Sigma t^2 x \qquad [15]$$

$$+ b_o \Sigma Y + b_1 \Sigma tY + b_2 \Sigma t^2 Y$$

$$+ c_o \Sigma xY + c_1 \Sigma txY + c_2 \Sigma t^2 xY$$

where

z = yield (bu)

x = precipitation (inches)

Y = temperature (^{o}F)

t = time interval (30 days used here = 1, 60 day = 2, etc.)

Ai, ai, bi, ci are regression constants.

Equation [15] does not allow for interaction between time periods and considers response to be a linear function of temperature and precipitation.

Applying regression analysis similar to Hendricks and Scholls, Runge and Odell (1958), using 53 years of Illinois corn yield data, found that 75% of the yield variability could be explained using precipitation and temperature data from 50 days before anthesis to 14 days after anthesis.

Leeper et al. (1974) developed an analysis model similar to that of Runge and Odell (1958) and Hendricks and Scholl (1943) except that they included a term for plant-available soil moisture content. From their analysis, they concluded that the greatest yield reduction occurred when moisture stress occurred within the 6 week period from 4 weeks before to 2 weeks after tasseling. They also concluded that the plant-available stored soil moisture was slightly less effective than was rooting depth or amount of preseason water available in the root zone in explaining variations in corn yield.

Stewart et al. (1975) working in California conclude that (1) when working with deep profile soils, a full soil water profile at the time of planting facilitates full and rapid development of the root system.

This reduced the possibility of the plant receiving rapid shock at some later time in the growing season as a result of water stress. (2) Pollination is an especially vulnerable time for the crop to experience a water deficit but conditioning with light deficits early in the season considerably reduces the susceptibility of the crop to drought at pollination. (3) Late irrigation, at the blister kernel stage or later always resulted in increased vegetative growth and increased evapotranspiration but had either no effect or a negative effect upon grain yield.

Huda et al. (1976) doing work in India with maize under monsoon conditions concluded that above average rainfall was only of value during the emergence of the plant. During the silking and tasseling to maturity stage, rainfall in excess of the average had a depressive effect upon the maize yield. From this study, it is not unreasonable to conclude that excess water can and does result in yield depression.

CHAPTER II

SECTION 2

METHODS AND MATERIALS FOR CROP MODEL

One of the objectives of this study was the development of a moisture dependent grain yield production function. The procedure of Jensen et al. (1973) was used to estimate the water content in the soil profile at any day in time.

The basic equations used are

$$\Theta_{i+1} = \Theta_i + \text{Precipitation} + \text{Irrigation} - ET_a - \text{Drainage} \qquad [16]$$

$$ET_a = K_c Et_p \qquad [17a]$$

where

ET_a = actual evapotranspiration/day (cm/day)

K_c = a dimensionless constant

$$ET_p = \Delta/(\Delta+\alpha)*(Rn+G)+\alpha/(\Delta+\alpha)*15.36* \qquad [17b]$$

$$(1.0+.0062*U_z)*(e_z^o - e_z)*C$$

where

Δ = Slope of the saturation vapor pressure-temperature curve de^o/dT (mb/^{o}c).

α = Psychrometric constant (mb/^{o}c).

Rn = Net radiation (cal/cm^{2}day).

G = Heat flux density from ground (cal/cm^2 day).

U_z = Horizontal wind speed at height z (km/day).

e_z^o = Saturation vapor pressure at height z (mb).

e_z = Actual vapor pressure at height z (mb).

C = Specific heat of vaporization (cm water/cal/cm^2).

ET_p = Potential evapotranspiration at height z (cm/day).

$K_c = K_{co} K_a + K_s$

where

K_{co} = the mean crop coefficient based on experimental data where soil moisture is not limiting. Parabolic splines (Kimbal,1976) were used to obtain the following growth curves.

0-40 days (K_{co} = .155473+.07741*day# after emergence + .000127*(day#)^2)

40-75 days (K_{co} = -.492238+.042127*day# -.00027*(day#)^2)

75-end of season (K_{co} = -.599704+.04493 *(day#)-.000297x(day#)^2)

The original curve was taken from Stegman (1977).

Ka = the relative crop coefficient related to available soil water.
(Ka = ln (Am+1)/ln(101))
where Am = % of available soil moisture present in the profile.

Ks = the increase in the crop coefficient when the soil surface is wetted by rain or irrigation.

Ks = (.9-Kc)*.8 for a day of rain or irrigation;

Ks = (.9-Kc)*.5 for a day after rain or irrigation;

Ks = (.9-Kc)*.3 for 2 days after rain or irrigation.

For the development of our model, we began by dividing the growing season into a set of 6 physiological stages of growth as found in Table II-1. Within each one of these stages, an attempt is made to determine the cumulative degree of stress to which the plants are subjected. The procedure implies that the maximum yield will occur as a result of minimum stress. Two functions that meet this criteria are

$$\text{Yield} = f\{\sum_{i=1}^{n} (\text{degree of stress})\} \qquad [18]$$

Table II-1. Physiological stages of growth used in crop model.

Stage # Physiological stage of growth data and cm of precipitation.	Approximate number of days in stage (126 day corn)	Phenological effects from water and water stress
(1) planting to 2 leaf May 10-May 20 .41 cm	planting-emergence-7 days after* emergence	In semi arid climates this stage of production is extremely critical for it has tremendous influence upon final plant population. There seems to be moisture threshhold below which little or no germination will occur.
(2) 2 leaf to 10 leaf May 20-June 9 4.10 cm	7 days-35 days*	During early period of the 2 leaf to 10 leaf stage the soil is cool and irrigation possibly could cool the root zone even further reducing nutrient availability to these young plants. Flooding for a period of several days during this stage when the growing point is below the soil surface can cause death of the corn plant.
(3) 10 leaf to silk emergence June 9-July 20 9.35 cm	35-66 days*	Moisture deficiencies during this stage markedly influence the growth and development of ears. This can result from length of ear and the number of possible kernels. Tassel emergence occurs during this stage of growth.
(4) silk emergence to beginning dough stage July 20-August 3 .31 cm	66-90 days*	Moisture stress during the pollination stage can cause poor fertilization and seed set resulting in substantial yield reduction. Blister formation which is in the middle of this stage begins a period of rapid grain weight increase. Moisture stress at this phase can reduce the rate of grain filling.
(5) beginning dough to full dent August 3-August 27 6.07 cm	90-114 days*	This is the final stage of grain filling. Unfavorable moisture conditions result in unfilled kernels and "chaffy" ears.
(6) full dent to maturity August 27-Sept. 21 2.70 cm	114-126 days*	Little or no increase in weights will result at this stage in the growing cycle.

*Day numbers were taken from Hanway (1966) and do not concur exactly with our dates.

or

$$\text{Yield} = f\{\prod_{i=1}^{n} (\text{degree of stress})\} \qquad [19]$$

where f{ } = yield function for season

i = 1 to n = the day number within each growth stage.

A large portion of the literature review dealt with the same kind of functional relationship that is represented above. Some important questions are: Is the function additive or multiplicative within stages of growth and between stages of growth? What functional relationship can best describe the term which we have labeled "degree of stress"? In theory, the multiplicative equation will show an interaction between different stages of growth better than an additive model. An example of interaction would be an extremely severe drought in the 10 leaf stage which kills the entire crop. The multiplicative model would put a zero into the yield equation resulting in zero total yield while the additive model would give positive results for the total cycle. With the arguments as above, the multiplicative version would seem to be the logical method to use; however, when working with average field conditions, extremes as mentioned above are seldom encountered. Experience has indicated that the additive model is a better predictor in some real field situations.

The degree of stress term can be written in several different ways. One of the most widely accepted ways is to characterize the term as being the difference between the evapotranspiration potential and the actual evapotranspiration raised to some power, i.e. $(ET_p-ET_a)^K$. The exponential term attributes a greater degree of effect to severe stress than to light stress compared with a linear (non-exponential) function. Although this relationship is one of the most acceptable methods of quantitatively describing stress, to determine the stress term becomes difficult due to problems in determination of the ET_a term. To do an accurate job of evaluation of the ET_a term would require the use of a weighing lysimeter which could accurately separate drainage terms from evaporation and transpiration components.

As an alternative to ET_a, we utilized a percent available moisture content term which is more easily determined. The term within each stage of growth looks like

$$MD = \{\sum_{i=1}^{N} (100-PAM)^2\}/N \qquad [20]$$

where

MD = moisture deficit term

PAM = Percent available moisture on day i is equivalent to (Θ for profile/Θ at field capacity)

N = total number of days in a particular stage of growth

To calculate MD, the method described earlier by Jensen et al. (1971) was used with corrections for readings taken with the neutron probe when that data was available. Table II-2 is an example of the output data from this program.

The yield function can be written

$$\text{Yield} = C+\sum_{i=1}^{N} \lambda_i MD_i \qquad [21]$$

or

$$\text{Yield} = C*\prod_{i=1}^{N} MD_i^{\lambda_i} \qquad [22]$$

where

N = number of growth stages in growing season.

Table II-2.

Calculated Soil Moisture Deficit During Each Stage of Growth

Plot	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6	Yield
11	84.0	20.0	218.0	69.0	200.0	586.0	113.3
18	91.0	23.0	190.0	215.0	432.0	940.0	119.8
21	25.0	2.0	95.0	91.0	240.0	652.0	113.7
8	57.0	9.0	32.0	66.0	191.0	560.0	130.4
17	68.0	12.0	32.0	69.0	200.0	586.0	133.9
26	40.0	4.0	32.0	97.0	394.0	872.0	97.4
19	40.0	3.0	95.0	51.0	81.0	364.0	128.1
23	81.0	20.0	90.0	60.0	67.0	319.0	108.4
24	58.0	9.0	191.0	164.0	54.0	294.0	106.4
9	136.0	47.0	222.0	69.0	200.0	586.0	114.6
13	291.0	149.0	309.0	164.0	357.0	832.0	133.1
14	63.0	10.0	97.0	232.0	633.0	1216.0	135.7
4	216.0	98.0	91.0	69.0	200.0	586.0	94.3
2	30.0	2.0	33.0	69.0	200.0	586.0	131.0
12	80.0	18.0	76.0	42.0	37.0	259.0	124.1
6	117.0	37.0	226.0	1562.0	2003.0	2866.0	105.4
15	24.0	2.0	91.0	910.0	775.0	1386.0	94.0
16	47.0	5.0	96.0	1049.0	1443.0	2222.0	100.7
3	85.0	2.0	73.0	69.0	200.0	586.0	116.7
5	72.0	2.0	73.0	69.0	200.0	586.0	98.9
22	33.0	2.0	65.0	63.0	183.0	536.0	102.7
1	203.0	2.0	195.0	1466.0	1900.0	2750.0	80.8
20	74.0	2.0	177.0	1305.0	1209.0	1943.0	72.5
25	83.0	2.0	195.0	1306.0	1018.0	1692.0	89.6
7	62.0	10.0	187.0	1432.0	1864.0	2708.0	111.2
10	45.0	4.0	170.0	1368.0	1794.0	2629.0	103.4
27	49.0	5.0	191.0	1293.0	500.0	1067.0	74.3

CHAPTER II

SECTION 3

RESULTS AND DISCUSSION OF CROP MODEL

The experimental plots discussed in Section I were designed so that results could be utilized for the derivation of an equation by the procedure of the previous section. The yield data found in Tables II-7, II-8, and II-9 showed that for winter wheat and spring wheat the F test indicated that there were no significant differences between treatments while the corn showed significant differences between treatments. The failure to show yield difference in the wheat crops can be attributed to the winter kill in the winter wheat and very timely rains falling on both spring and winter wheat.

For the corn plots at the James Valley Research and Extension Center, production functions and their developmental statistics which utilizes the procedure of the last chapter are shown in Tables II-3, II-4, II-5 and II-6. Looking first at the correlation of both the additive and multiplicative model, we see almost no correlation between stage 1 and yield. The results indicate that if early season field conditions are such that the plant population is relatively constant; then the amount of water in the profile has little effect on yield. There is a bias within our data base in that even the driest plots in the first stage of growth were near field capacity at the surface. In fact, this comment with respect to our data is applicable to the first three stages of growth. The correlation between the moisture deficit at the second stage of production with

Table II-3. Correlation Coefficients for Yield and Growth Stages Moisture Deficits

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Stage 2	.85					
Stage 3	.55	.46				
Stage 4	.01	-.20	.46			
Stage 5	.09	-.14	.40	.91		
Stage 6	.09	-.13	.40	.92	1.00	
Yield	0.00	.22	-.22	-.62	-.42	-.43

Table II-4.

Step-wise Linear Regression for (yield=$C+\sum_{i=1}^{6} \lambda i$ (moisture deficit))

C	λ_4	λ_5	λ_3	λ_1	λ_2	λ_6
118.03	-.0188					
SE(Y) = 14.2	MCC = .624	F = 15.96**				
115.17	-.0433	.0245				
SE(Y) = 12.8	MCC = .721	F = 6.519*				
112.78	-.0449	.0247	.0230			
SE(Y) = 13.0	MCC = .726	F = $.358^{NS}$				
113.72	-.0496	.0276	.0556	-.0558		
SE(Y) = 12.99	MCC = .741	F = 1.082^{NS}				
117.41	-.0467	.0283	.0387	-.1446	.2209	
SE(Y) = 12.74	MCC = .766	F = $.598^{NS}$				
124.75	-.0451	.0562	.0373	-.1493	.2356	-.0223
SE(Y) = 12.99	MCC = .768	F = $.186^{NS}$				

Table II-5. Correlation Coefficients for Log Yield and Log Growth Stages Moisture Deficit

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
Stage 2	.67					
Stage 3	.48	.32				
Stage 4	.03	-.20	.54			
Stage 5	.07	-.18	.37	.88		
Stage 6	.08	-.17	.40	.90	1.00	
Yield	-.03	.35	-.32	-.61	-.45	-.46

Table II-6.

Step-wise Regression for $(\text{yield}=C^* \prod_{i=1}^{6} (\text{moisture deficit})^{\lambda i})$

C	λ4	λ2	λ1	λ6	λ5	λ3
2.208	-.076					
SE(Y) = .061	MCC = .607		F = 14.6**			
2.166	-.070	.032				
SE(Y) = .059	MCC = .652		F = 2.31			
2.295	-.063	.064	-.095			
SE(Y) = .057	MCC = .696		F = 2.71			
2.056	-.130	.069	-.111	.143		
SE(Y) = .055	MCC = .741		F = 3.13†			
1.264	-.162	.068	-.119	.745	-.349	
SE(Y) = .054	MCC = .765		F = 1.85			
1.23	-.171	.066	-.125	.760	-.353	.016
SE(Y) = .055	MCC = .766		F = .07			

†Significant at .9 level.

Table II-7. 1977 Spring Wheat Yield
bu/acre (kg/hectare)

Treatment	1st rep	2nd rep	3rd rep	Mean
Fall	34.9 (2063)	31.7 (2130)	29.8 (2002)	32.1 (2157)
Fall and boot	35.3 (2372)	32.0 (2150)	31.4 (2110)	32.9 (2211)
Boot	32.9 (2210)	33.7 (2264)	37.9 (2546)	34.8 (2340)
Joint boot*	33.4 (2244)	27.9 (1874)	32.3 (2170)	31.2 (2097)
Fall, boot*, joint	30.7 (2063)	37.3 (2507)	34.5 (2318)	34.2 (2297)
Dryland	39.0 (2003)	38.7 (2601)	34.6 (2325)	37.4 (2513)

*Because of excess rain the boot stage was not irrigated.

Analysis of Variance

Source	Degree of freedom	Sum of squares	Mean squares	F
Treatment	5	74.2	14.8	2.04
Error	12	87.5	7.3	
Total	17	161.8		

F not significant at .05 level.

Standard error within treatments 1.56
Standard error between treatments 2.21
Coefficient of variation 8.0%

Using (lsd) the fall treatment and the joint-boot treatment were significantly different from dryland at the .05 level.

Table II-8. 1977 Winter Wheat Yield Analysis
bu/acre (kgm/ha)

Treatment	1st rep	2nd rep	3rd rep	Mean
Fall	31.90 (2144)	31.00 (2083)	27.80 (1868)	30.23 (2031)
Joint	23.10 (1552)	40.70 (2735)	36.10 (2426)	33.30 (2238)
Dryland	21.60 (1452)	30.80 (2070)	23.80 (1599)	25.40 (1707)
Fall	28.90 (1942)	30.30 (2036)	23.50 (1579)	27.57 (1853)
Fall, joint	32.40 (2177)	33.60 (2258)	30.60 (2056)	32.20 (2164)
Joint	31.90 (2144)	31.40 (2110)	38.10 (2560)	33.80 (2271)
Dryland	21.00 (1411)	28.80 (1935)	32.50 (2184)	27.43 (1843)

*Because of excess rain boot irrigation was not performed.

Analysis of Variance

Source	Degree of freedom	Sum of squares	Mean squares	F
Treatment	6	191.678	31.946	1.281
Error	14	349.220	24.944	
Total	20	540.898		

F not significant at .05 level.

Standard error from treatment mean 2.884
Standard error of difference between treatment means 4.078
Coefficient of variation 16.7%

No treatments were found different from dryland used (lsd) at .05 level.

Table II-9. 1977 Corn Yield
bu/acre (kg/hectare)

Treatment	1st rep	2nd rep	3rd rep	Mean
Fall	105.4 (6619)	100.7 (6324)	94.0 (5903)	100.0 (6280)
Spring post plant	72.5 (4553)	89.6 (5627)	80.8 (5074)	81.0 (5087)
Fall and tasseling	119.8 (7523)	113.3 (7115)	113.7 (7140)	115.6 (7260)
Spring and tasseling	128.1 (8045)	106.4 (6682)	108.4 (6808)	114.3 (7178)
Tasseling	133.1 (8359)	114.6 (7197)	135.7 (8522)	127.8 (8026)
Fall, 12 leaf, and silking	97.4 (6117)	133.9 (8409)	130.4 (8189)	120.6 (7574)
Spring, 12 leaf and silking	98.9 (6211)	116.7 (7329)	102.7 (6450)	106.1 (6663)
12 leaf and silking	124.1 (7793)	94.3 (5922)	131.0 (8227)	116.5 (7316)
Dryland	74.3 (4666)	103.4 (6494)	111.2 (6983)	96.3 (6048)
			Mean	108.7 (6826)

Analysis of Variance

Source	Degree of freedom	Sum of squares	Mean squares	F
Treatment	8	4949.05	618.63	3.38*
Error	18	3295.03	183.06	
Total	26	8244.08		

Standard error within treatments 7.811
Standard error between treatments 11.047

Using (lsd .05) fall-tasseling, tasseling, fall-12 leaf-silking, and 12 leaf-silking treatments are significantly different from dryland.

Using (lsd .01) tasseling treatment is significantly different from dryland.

yield is positive. This result indicates that our assumption that a maximum soil moisture content will produce a maximum yield is invalid. In fact, the logical conclusion is that some type of waterlogging occurred which caused stress possibly as a result of oxygen deficiency. Another possible effect of the wet soil condition is that the plants in the wetter soils may not have developed as deep of root system as some of the other plants growing under drier conditions. This would result in the plants in the drier soil having the advantage of a deeper root zone for later absorption of moisture and nutrient. There is also the possibility that excessive moisture can cause early season nitrogen leaching.

Stage 4 shows the highest correlation with yield. The literature review gave substantial evidence to support the conclusion that the most critical time for stress is the period around tassel and silk emergence. The correlation coefficient of -0.62 for the additive model and -0.61 for the multiplicative model is a valid indication of the importance of stress in stage 4.

In our analysis, the 5th and 6th growth stages could have been considered as a single stage since the correlation coefficient between 5 and 6 is essentially 1.0. These two growth periods should have been differentiated by an irrigation but heavy rains during the proposed time resulted in elimination of the planned irrigation. With a correlation of -0.42 and -0.46 for 5 and 6, respectively, these periods have the second highest correlation with yield.

From the data analyzed, the additive multiple linear regression model, Table II-4, recognizes only two of the variables as being significant contributors to the crop yield equation. The wet preplant and early postplant weather resulted in very little early season differences in the soil moisture profiles and thus stages 1, 2 and 3 showed little difference in effect upon the production model. The first variable selected by the regression was the stage 4 term while the second term was the stage 5 term. As a result of the interaction between the two terms, the coefficient is positive. One can only interpret this result to mean that a greater soil water deficit in the 5th period produces greater yield which certainly is not true in all cases. As an example of how absurd this equation can be, if the soil profile in stage 4 was at field capacity and if throughout stage 5 the profile was at permanent wilting point the model would predict a yield of 355 bu/acre.

The stepwise regression analysis for the multiplicative version of the model, Table II-6, selected the 4th stage variable as most significant. The analysis then selected the second, first, and sixth variable in that order with only the sixth variable significant at the 0.9 level. The multiplicative version utilized four variables until it failed to show any significant improvement. Excess moisture at stage two accounts for the positive sign of this coefficient. The positive value of the coefficient of the sixth can be attributed to the high correlation between stage 5 and stage 6.

Comparing the two models, it is difficult from the limited data base to determine which model will yield better results. From Tables II-4 and II-6 we can see that the standard error of the additive equation is less than the standard error of the multiplicative version even though the multiplicative version utilized more significant variables. This by itself would indicate that, as Hanks (1974) concludes, the additive model seems to predict better than the multiplicative version.

There are both advantages and disadvantages in utilizing a soil moisture depletion term as opposed to an actual ET term. As discussed earlier, the reason for using a stress term was the feeling that with the use of corrections from measured data one could improve upon the accuracy of ET data alone.

In theory, the biggest disadvantage with a depletion term is the variability in energy input into the crop growing system. Energy input varies from year to year and no doubt is significantly different some years. It was our feeling that other variables such as water management, fertilizer management, and pest management were of considerably greater magnitude and thus dwarf the problem of variation of input energy.

Our results indicate that at certain periods within the growing season for crops grown on relatively fine textured soils with deep profiles there is a real possibility of yield depression resulting from excess precipitation and/or irrigation. This was especially true with our spring irrigation which showed a negative correlation with yield and contributed significantly to the regression analysis with a negative coefficient in the stepwise regression equation. The fall irrigation treatment correlated positively with yield but the magnitude was quite small. The fall irrigation term was not significant when added to the regression analysis and thus could not be considered a significant contributor to yield. As would be expected, the tassel-silking stage showed the greatest correlation with yield and contributed the largest coefficients in the regression analysis procedure.

CHAPTER III

SECTION 1

INTRODUCTION AND LITERATURE REVIEW FOR SOIL WATER MOVEMENT SIMULATION

In the last two chapters we have discussed the very practical questions of water loss over winter and yield response of a corn crop to water availability at different stages of physiological maturity.

An important objective of physical scientists working with soil water movement is to solve the theoretical moisture flow equation for practical field situations using any soil type, weather, or crop cover condition. When this objective has been satisfied, many problems such as water loss over winter could be solved with a mathematical simulation procedure.

This chapter contains theoretical background and a solution of the basic flow equation for a field profile using basic upper and lower boundary conditions. This solution provides a base for the analysis of more complex situations.

Early philosophers believed that at such time as they were able to understand and empirically define the principals of the universe, they could predict physical phenomena. In 1856 Henry Darcy, looking at analogies in the area of thermodynamics, suggested that the quantity of water flowing through a sand column of length L, and cross sectional area A, during a time t, could be empirically described by the equation

$$\frac{Q}{t} = -KiA = -K\frac{\Delta h_t}{L}A \qquad [23]$$

or $\frac{Q}{At} = -K\frac{\Delta h_t}{L} \rightarrow q = -K\frac{\Delta h_t}{L}$

where

Q = quantity of water (cm^3)

t = time (day)

K = hydraulic conductivity (cm/day)

$i = \Delta h_t/L$ = gradient (cm/cm)

h_t = total head or total hydraulic potential (cm)

L = length (cm)

A = area (cm^2)

q = flux (cm^3/cm^2 day)

Bernoulli presented a theorem for pure water which stated that the total hydraulic potential, h_t, is composed of three components, the gravity head, the pressure head and the velocity head. This equation may be written as

$$h_t = z + \rho/\alpha + \gamma^2/2g = z + \Psi + V \quad [24]$$

where z = elevation or positional head (cm)

$\rho/\alpha = \Psi$ = pressure of fluid/specific weight ($gm/cm^2/gm/cm^3$)

$\gamma^2/2g = V$ = velocity/acceleration due to gravity ($cm^2/sec^2/cm/sec^2$)

For water containing salts an additional term for osmotic potential must be added. Because of the small order of magnitude of the velocity and osmotic potentials they are not considered significant so the equation for total head is assumed to be

$$h_t = z + \Psi \quad [25]$$

Next we look at a simple parallelepiped and consider the unsteady state of a noncompressible fluid flowing into and out of the parallelepiped. If we assume the system is dynamic in only one direction we must have conservation of mass in that direction and so

$$\text{Inflow-outflow} = \overset{\text{inflow}}{\{g_x \Delta y \Delta z\}} - \overset{\text{outflow}}{\{[g_x + \frac{\partial g_x}{\partial x}\Delta x]\Delta y \Delta z\}} \qquad [26]$$

$$= -\frac{\partial g_x}{\partial x}\Delta x \Delta y \Delta z$$

where Δx, Δy, Δz = directional components (cm)

g_x = flux term in the x direction (cm^3/cm^2 day = cm/day)

An even more general case of the above which will include compressible fluids can be obtained by multiplying through by the fluid density ρ so we have

$$\text{Inflow-outflow} = \frac{-\partial \rho g_x}{\partial x}\Delta x \Delta y \Delta z \qquad [27]$$

where ρ = density of fluid (gm/cm^3)

Now if we would allow for flow in all three directions we could have

$$\text{Inflow-outflow} = -\{\frac{\partial \rho g_x}{\partial x} + \frac{\partial \rho g_y}{\partial y} + \frac{\partial \rho g_z}{\partial z}\}\Delta x \Delta y \Delta z \qquad [28]$$

In a porous medium such as soil, and considering a time dependent situation, we can say that

$$\text{Inflow-outflow} = \frac{\partial(\rho \Theta)}{\partial t}\Delta x \Delta y \Delta z \qquad [29]$$

where Θ = volumetric moisture content (cm^3 water/cm^3)

Combining equations [28] and [29] we get

$$\frac{\partial(\rho\Theta)}{\partial t} = -\{\frac{\partial \rho g_x}{\partial x} + \frac{\partial \rho g_y}{\partial y} + \frac{\partial \rho g_z}{\partial z}\} \qquad [30]$$

If our only concern is with flow in one direction and if we assume ρ to be held constant, then we obtain the equation

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial g_z}{\partial z} \qquad [31]$$

Substitution of equation [23] into [31] gives

$$\frac{\partial\Theta}{\partial t} = \frac{\partial\{K\frac{\partial h_t}{\partial z}\}}{\partial z} \qquad [32]$$

Substition of equation [25] into [32] gives

$$\frac{\partial\Theta}{\partial t} = \frac{\partial\{K\frac{\partial(\Psi+z)}{\partial z}\}}{\partial z} \qquad [33]$$

Researchers involved in development of computer models to simulate field situations have been concerned with the problem of soil water hysteresis.

Dane and Wierenga (1975) working with a clay loam and Vachaud and Thony (1971) working with sand both concluded that utilization of the main wilting or main drying portion of the Ψ vs Θ family of curves was inferior to using a scanning curve. A scanning curve will be a uniquely determined curve with origin at the moisture content of the transition point from which wetting to drying or drying to wetting took place. The mathematics for determination of the scanning curve can be handled as by Mualem (1973).

Rose (1971) working with sand, loam, and clay and Vachaud and Thony (1971) concluded that hydraulic conductivity, K, is very nearly a single-valued function of the moisture content Θ. Because of the hysteresis of the Ψ vs Θ curve and assuming that Rose, Vachaud and Thony were correct then K cannot be considered a single valued function of Ψ.

Cary (1975) working with a silty clay, a loamy sand, and a silt loam addressed the problem of temperature and pressure variations. He cycled soils through temperature changes of from 5°C to 25°C and found that, for practical purposes, normal temperature and pressure changes will not significantly affect the normal hysteresis of the Ψ vs Θ curve.

If equation [33] is to be of value it is necessary to develop it into an equation with a common independent variable. To do this Θ must be expressed as a function of Ψ or

$$\Theta = f(\Psi) \text{ and } \Psi = f(\Theta) \qquad [34]$$

Unfortunately for most soils these functions are not single-valued because of hysteresis which varies with depth and time. For practical field problems we assumed that variation from site to site within field is greater than hysteresis effects and essentially ignore the hysteresis problem.

If we assume [34] to be single-valued and

$$D = K \frac{\alpha\Psi}{\alpha\Theta} \qquad [35]$$

where D = deffusivity (cm^2/day)

then from [33] and [34]

$$\frac{\partial \Theta}{\partial t} = \frac{\partial \{D(\Theta) \frac{\partial \Theta}{\partial z} - K(\Theta)\}}{\partial z} \qquad [36]$$

Now that we have developed an equation for moisture flow it becomes necessary to solve this equation.

There are three known methods for obtaining solutions to partial differential equations. The first and most obvious is an analytical solution to the particular problem. Because of the nonlinearity of the equations, in order to obtain a solution it is necessary to make assumptions, some of which might be that the system is steady state and the entire soil profile is homogenous. This type of simplification results in a solution that is of little value for actual field situations. The second method is utilization of an electrical analog to simulate moisture flow with electricity. Justification for this method is derived from the similarity between Darcy's law [23] and Ohm's law

$$I = E/R \tag{37}$$

where

I = current flow (amps)

E = electrical potential (volts)

R = resistance to electrical flow (ohms)

and if we substitute

$$R = R'\frac{L}{A} \tag{38}$$

where

R' = specific resistance (ohm cm)

L = length (cm)

A = area (cm^2)

and

$$R' = \frac{1}{K_E} \tag{39}$$

where

K_E = electrical conductivity ($\frac{1}{\text{ohm}}$ cm)

we now have the equation

$$I = \frac{K_E EA}{L} \tag{40}$$

which appears to be very similar to equation [23] in the form

$$\frac{Q}{t} = \frac{K\Delta h_t A}{L} \tag{41}$$

By building an actual resistance network, simulation of a one- or two-dimensional system can be obtained. This method is slow and laborious but it is capable of simulating a non-steady state system (Hoover, 1975).

Because of the non-availability of a universal analytical solution and the lack of flexibility of the analog type solution, many researchers have turned to numerical methods to solve the flow equations.

The numerical procedure usually begins with the Taylor series about x, first in the forward direction and then in the backward direction (Remson, Hornberger, and Molz, 1971).

$$f(x+\Delta x) = f(x)+\Delta x\frac{df}{dx} + \frac{(\Delta x)^2}{2!}\ \frac{d^2f}{dx^2} \ldots \qquad [42]$$

$$f(x-\Delta x) = f(x)-\Delta x\frac{df}{dx} + \frac{(\Delta x)^2}{2!}\ \frac{d^2f}{dx^2} \ldots \qquad [43]$$

Subtracting these two equations and solving for $\frac{df}{dx}$ we get the central difference approximation which is

$$\frac{df}{dx} \simeq \frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x} \simeq \frac{f(x+(1/2)\Delta x)-f(x-(1/2)\Delta x)}{\Delta x} \qquad [44]$$

Using a similar analysis we solve for $\frac{d^2f}{dx^2}$ which gives

$$\frac{d^2F}{dx} \simeq \frac{f(x+\Delta x)-2f(x)+f(x-\Delta x)}{\Delta x^2} \qquad [45]$$

Equations [44] and [45] have truncation errors caused by dropping the terms to the right of where $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ appear in the Taylor expansion.

Richtmyer and Morton (1967) discuss 14 implicit finite difference methods for the simple heat flow problem. He describes the general equation for finite differences as

$$\frac{\partial f}{\partial t} = \frac{f_j^{n+1} - f_j^n}{\Delta t} \qquad [46]$$

$$\frac{\partial^2 f}{\partial x^2} = (\alpha)\frac{(f_{j+1}^n - 2f_j^n + f_{j-1}^n)}{\Delta x^2} + (1-\alpha)\frac{(f_{j+1}^{n+1} - 2f_j^{n+1} + f_{j-1}^{n+1})}{\Delta x^2} \qquad [47]$$

When $\alpha = 1/2$ we have the Crank and Nicholson (1947) method and when $\alpha = 0$ we have an explicit system where we find f_j^{n+1} directly in terms of the known quantities f_j^n. When $\alpha \neq 0$ we develop a system of simultaneous linear equations to find f_j^{n+1} and the system is called implicit.

Since the development and use of digital computers, numerical methods for solutions to flow equations have become practical. A review of the work on one dimensional flow is in order at this time.

Klute (1952) looked at the general horizontal flow equation.

$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial x}\{\frac{\partial \Theta}{\partial x}\} \qquad [48]$$

Utilizing the Boltzmann Transformation, he provided the initial workable solution to the flow equation. After transforming the partial differential equation to a non-linear ordinary differential equation it is converted into a solvable equation. Some of the problems involved include a limitation of the problem to uniform initial moisture conditions and utilization of a semi-infinite uniform medium. It is interesting that Klute foresaw the use of the Crank and Nicholson solutions but did not in this early paper utilize this procedure. Klute's paper was followed by Staple and Lehane (1954) Day and Luthin (1956) and Gardner (1959) utilizing Beltzmann Transformation procedures.

Willis (1960) utilized an analytical and graphical solution to the Darcy equation with a gravity term to describe steady-state water movement in a layered situation.

The next significant advancement of the art was developed in 1962 (Hanks and Bowers, 1962). They solved the basic flow equation utilizing the Crank and Nicholson finite difference approach. The flow equation is linearized with a predictive value for K (Θ) and D (Θ). They also suggest in this paper a method of determining a dynamic Δt where

$$\Delta t^{j+1/2} = Q/I^{j-1/2} \qquad [49]$$

where Δt^{j+1} = the time period for the next cycle

Q = a constant of water entering the soil which equals .035Δx (cm)

$I^{j-1/2}$ = infiltration rate from the previous time (cm/day)

In addition they suggest a predictive term for Θ_i^{j+1} to be utilized to estimate K (Θ) where

$$\Theta_i^{j+1} = (\Theta_i^j - \Theta_i^{j-1})\beta + \Theta_j^j \qquad [50]$$

where

β = a constant 0.7 or $\dfrac{\Delta t^{i+1}}{(\Delta t^{i}+3.3)}$

whichever is greater

There have been in the literature a large number of papers on the subject of finite difference solutions to the one dimensional vertical flow equation. The majority of them utilize the Crank and Nicholson approach or the more generalized form of this equation as described by Richtmyer and Morton (1967). Table III-1 lists a summary of some of the more significant finite difference papers.

In the root zone of a field under natural conditions, we find many factors contributing to moisture movement. One important factor which

Table III-1. Summary of finite difference equations.

Researcher	Equation solved	Method
Ashcroft et al 1962	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} D(\theta) \frac{\partial \theta}{\partial x}$	forward finite difference implicit tridiagonal solution
Rubin and Steinhardt 1963	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \{D(\theta) \frac{\partial \theta}{\partial x} - K(\theta)\}$	central finite difference implicit matrix solution
Green et al 1964	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \{K \frac{\partial h_t}{\partial x}\}$	Hanks and Bowers
Staple 1966	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} K \frac{\partial \Psi}{\partial x} + \frac{\partial k}{\partial z}$	backward finite difference explicit solution
Liakopoulos 1966	$\frac{\partial \Psi}{\partial t} = D(\Psi) \frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial k}{\partial \theta} \frac{\partial \Psi}{\partial z} \{\frac{\partial \Psi}{\partial z} + 1\}$	central finite difference implicit tridiagonal solution
Remson et al 1967	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} D \frac{\partial \theta}{\partial z} + \frac{\partial k}{\partial z}$	backward finite difference explicit solution
Rubin 1967	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} (K \frac{\partial \Psi}{\partial \theta} \frac{\partial \theta}{\partial x} - K)$	central finite difference implicit matrix solution
Wang and Lakshminarayana 1968	$\frac{\partial \theta}{\partial t} = K \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial k}{\partial z} \{\frac{\partial \Psi}{\partial z} - 1\}$ $+ \frac{\partial \theta}{\partial z} \{\frac{\partial k}{\partial \theta} [\frac{\partial \Psi}{\partial z} - 1] + \frac{\partial k}{\partial z} \frac{\partial \Psi}{\partial \theta}$ $+ \partial K \frac{\partial^2 \Psi}{\partial \theta \, \partial z} + K \frac{\partial \Psi}{\partial \theta} \frac{\partial^2 \theta}{\partial z^2}\}$	implicit-explicit finite difference
Freeze 1969	$\frac{\partial \Psi}{\partial t} = \frac{1}{C(\Psi)} \frac{\partial}{\partial z} K(\Psi) (\frac{\partial \Psi}{\partial z} + 1)$	central finite difference implicit matrix solution
Hanks et al 1969	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (K(\theta) \frac{\partial h_t}{\partial z})$	Hanks and Bowers
Guymon 1970	$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} = C \frac{\partial}{\partial x} (\frac{D}{C} \frac{\partial \theta}{\partial x})$	finite element
Whisler et al 1972	$\frac{\partial h}{\partial t} = \frac{1}{C(h_z)} \frac{\partial}{\partial z} K(h_z) \frac{\partial h_z}{\partial z} + \frac{\partial K(h_z)}{\partial z}$	finite difference implicit matrix solution

must be considered is moisture transport due to thermal gradients. Philip and deVries (1957) attempted to explain moisture movement by dividing it into liquid and vapor movement. They derive their approach from the mechanics of fluids, the diffusion of mass, and the conduction of heat.

Taylor and Cary (1964) worked with a second approach based upon the thermodynamics of irreversible processes.

Cary (1965, 1966) discussed the phenomena of moisture movement in the liquid phase as a result of thermal gradients. He indicates four possible methods of inducing moisture flow due to thermal gradients. 1. Moisture flow in unsaturated soil from warm to cool could occur as a result of a surface tension gradient. 2. Moisture flow could occur from cool to warm resulting from the difference in specific heat content between the liquid layer adsorbed on the solid surface and the specific heat content of the bulk liquid in the pores. 3. Water movement induced by motion created by random kinetic energy changes associated with hydrogen bond distribution. 4. Moisture flow from thermally induced osmotic gradients. None of these relationships are well understood and Cary suggests that it is necessary to resort to a phenomenological approach to describe this type of flow. He suggests an equation of the form

$$Je = \frac{KQ}{aT}\frac{dT}{dz} \qquad [51]$$

where

Je = thermally induced liquid phase of flow (cm/day)

K = conductivity (cm/day)

Q = liquid phase heat of transport (ergs/gm)

a = acceleration of gravity (cm/sec^2)

z = vertical depth (cm)

T = temperature (°K)

The problem of separating the thermal conductivity from the isothermal capillary conductivities is difficult. When the K values are determined in situ, this method has an inherent inability to maintain a steady state thermal regime. In actuality the in situ method adds the thermal and capillary conductivites together. For this reason many authors (Harlan, 1973), (Kennedy and Lielmezs, 1973), (Guyman and Luther, 1974) and (Outcalt et al., 1975) have chosen to ignore thermal effects except for that which directly results from freezing. These were the same conclusions of Gurr et al. (1952) and Kuzmak and Sereda (1957).

The upper boundary or, more specifically the flux out of the top of the soil profile, has been an important question to soil physicists for many years. Rose (1966) and Rosenberg et al. (1968) give good summaries of the different approaches and varied attempts to describe actual evapotranspiration. The equation [17b] which was used here was that of Jensen et al. (1969) as described earlier.

CHAPTER III

SECTION 2

METHODS AND MATERIALS FOR SOIL WATER MOVEMENT SIMULATION

With the mathematic descriptions of soil moisture characteristics and with an adequate description of the upper and lower boundaries of the soil profile, it is possible to simulate the flow of moisture through a soil profile. The moisture flow through a field profile was simulated using a Hewlett Packard 9825A programmable calculator. To simulate a day of real time required about two hours of calculator time. The programmable calculator, although somewhat slower than an IBM 370-40 computer used earlier, proved to be an economical and practical machine for performing the mathematical analysis.

The study examined the flow of water through the profile of a Great Bend silt loam soil with simple boundary conditions.

The field portion of the study was conducted at the James Valley Research and Extension Center using three of the spring wheat plots mentioned earlier. The plots were fall irrigated and then covered with black plastic. The plastic cover established an upper boundary condition of essentially zero water flux. Since the work was performed in the fall, solar radiation absorption and the greenhouse effect of the plastic covered surface was minimized. As was noted earlier, water content measurements in the profile were taken with a neutron probe while surface water content was determined from gravimetric samples.

The simulation program requires input of the relationships of soil water hydraulic conductivity vs volumetric soil moisture content,

(K vs Θ), soil water diffusivity vs volumetric soil moisture content, (D vs Θ), and soil moisture tension vs volumetric soil moisture content, (Ψ vs Θ). Data to describe these input relationships were taken from Stone (1973) and Frankenstein (1973) and developed into subroutines so as to be easily accessible to the simulation program.

The Ψ vs Θ data for the Great Bend soil used in this study were collected by Stone (1973) with the exception of the 15 atm values which came from Frankenstein (1973). The datawere converted into a subroutine which sorts across 9 depths with 3 least square linear exponential equations per depth. The equations used were of the form

$$\ln \Psi = C_1 + C_2\Theta \qquad [53]$$

where C_1 and C_2 are regression constants

When the depth of the node being considered in a calculation is between any two of the 9 depths used in the subroutine, linear interpolation between the depths is used. This method would be invalid in a soil profile whose texture breaks off abruptly from one depth to the next but since the Great Bend soil is a uniform silt loam texture in the top 150 cm this assumption is acceptable.

In our attempts at obtaining reasonable functions to characterize existing Ψ vs Θ data sets several procedures were used including cubic and parabolic spline procedures and a linear interpretation procedure from a data table. As was stated earlier, we used a series of linear exponential equations for each depth. Figure III-1 is a comparison between Ψ vs Θ curves developed by Stone and the values used in the analysis. Although this method has some drawbacks it appears to be adequate.

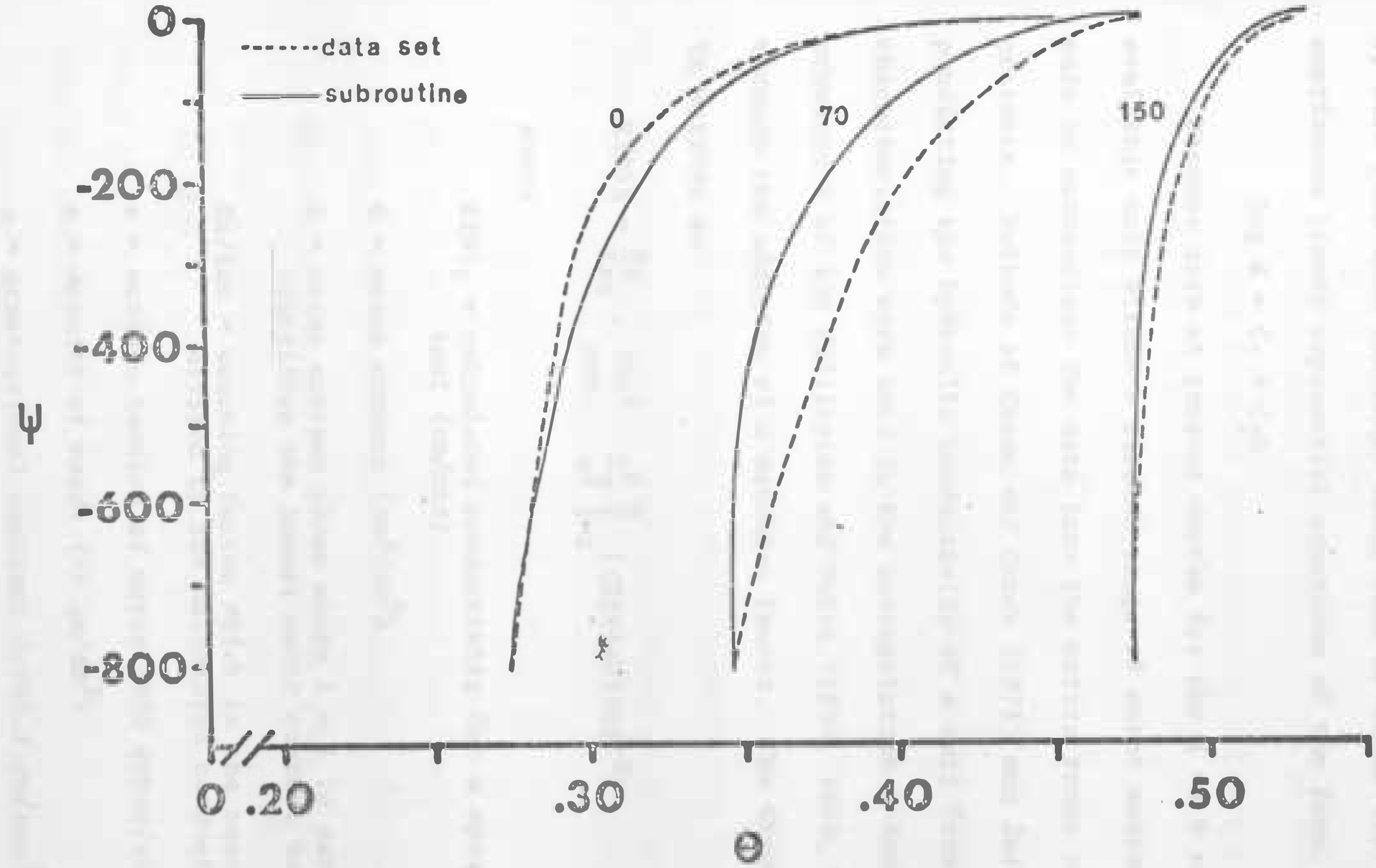


Figure III-1. Laboratory determined and subroutine generated Ψ vs Θ curves.

The hydraulic conductivity vs water content relationships, K vs θ, were generated initially using data of Stone (1973) and using two continuous linear exponential equations of the form.

$$\log K = C_1 + C_2\theta \qquad [54]$$

Because data at greater depths for the K vs θ relationship were available only within a limited range of water contents, attempts were made to extrapolate the data into the entire range of available water contents. Methods of Green and Corey (1971) and Jackson (1972) for predicting the hydraulic conductivity of a soil from moisture release characteristics were used in the extrapolation. Both methods are improvements of the Millington and Quirk (1959, 1960, 1961) equation through the addition of a matching factor. The Green and Corey equation is written as

$$K(\theta)i = \frac{Ks}{Ksc} \cdot \frac{30\alpha^2}{\rho g n} \cdot \frac{\varepsilon^P}{n^2} \sum_{j=i}^{m} [(2j+1-2i)hj^{-2}] \qquad [55]$$

where

$K(\theta)_i$ = calculated conductivity for a specified water content (cm/min)

θ = water content (cm^3/cm^3)

i = water content class where i = 1 is saturation and i = m identifies the lowest water content to be calculated

Ks/Ksc = matching factor which is the measured conductivity divided by the calculated conductivity

α = surface tension of water (≈70 dynes/cm)

ρ = density of water (≈1 gm/cm^3)

g = gravitational constant (≈980.7 cm/sec^2)

η = viscosity of water (≈.01 gm/sec cm)

$\varepsilon = \Theta$ saturation (cm^3/cm^3)

p = a parameter accounting for interaction of pore classes (1 is used here)

n = total number of pore classes to be considered

hj = the pressure or Ψ value for a given Θ

Jackson's equation is written as

$$K(\Theta)i = Ks(\varepsilon_i/\varepsilon_1)^p * \frac{\left(\sum_{j=i}^{m} [2j+1-2i]hj^{-2}\right)}{\sum_{j=1}^{m}[2j-1]hj^{-2}} \quad [56]$$

Although Jackson's paper was written more recently than the Corey and Green paper, the Corey and Green procedure gave results which agreed more closely with Stone's experimental results in the range where experimental data were available. Figure III-2 shows the curves generated by Corey and Green method compared with Stone's work.

Diffusivity terms (D) were calculated as a function of water content (Θ) by the equation

$$D(\Theta) = \frac{\partial\Psi}{\partial\Theta} K(\Theta) = \frac{\Psi(\Theta+.001)-\Psi(\Theta-.001)}{.002} K(\Theta) \quad [57]$$

Figure III-3 is a flow diagram of the simulation program used.

Block 1 set initial values and dimension variables.

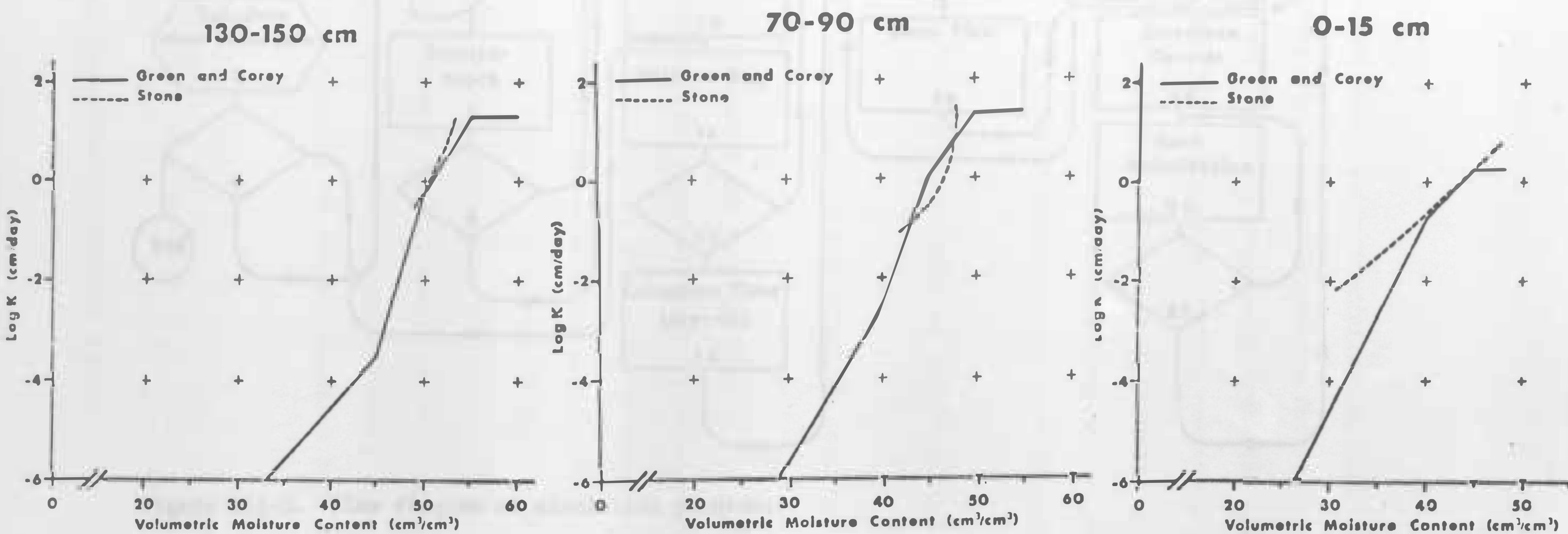
Block 2 set initial time increments, flux rate, and K_{sat}.

Block 3 calculates new conductivity and diffusivity matrices.

Block 4 is a decision block which determines if the program is going to go to a new day, not finished with present day or finished with the program.

Block 5 loads daily data necessary for the program into the computer. Variables are solar radiation, maximum and minimum daily

Figure III-2. Comparison of Green and Corey vs Stone (Ψ vs Θ).



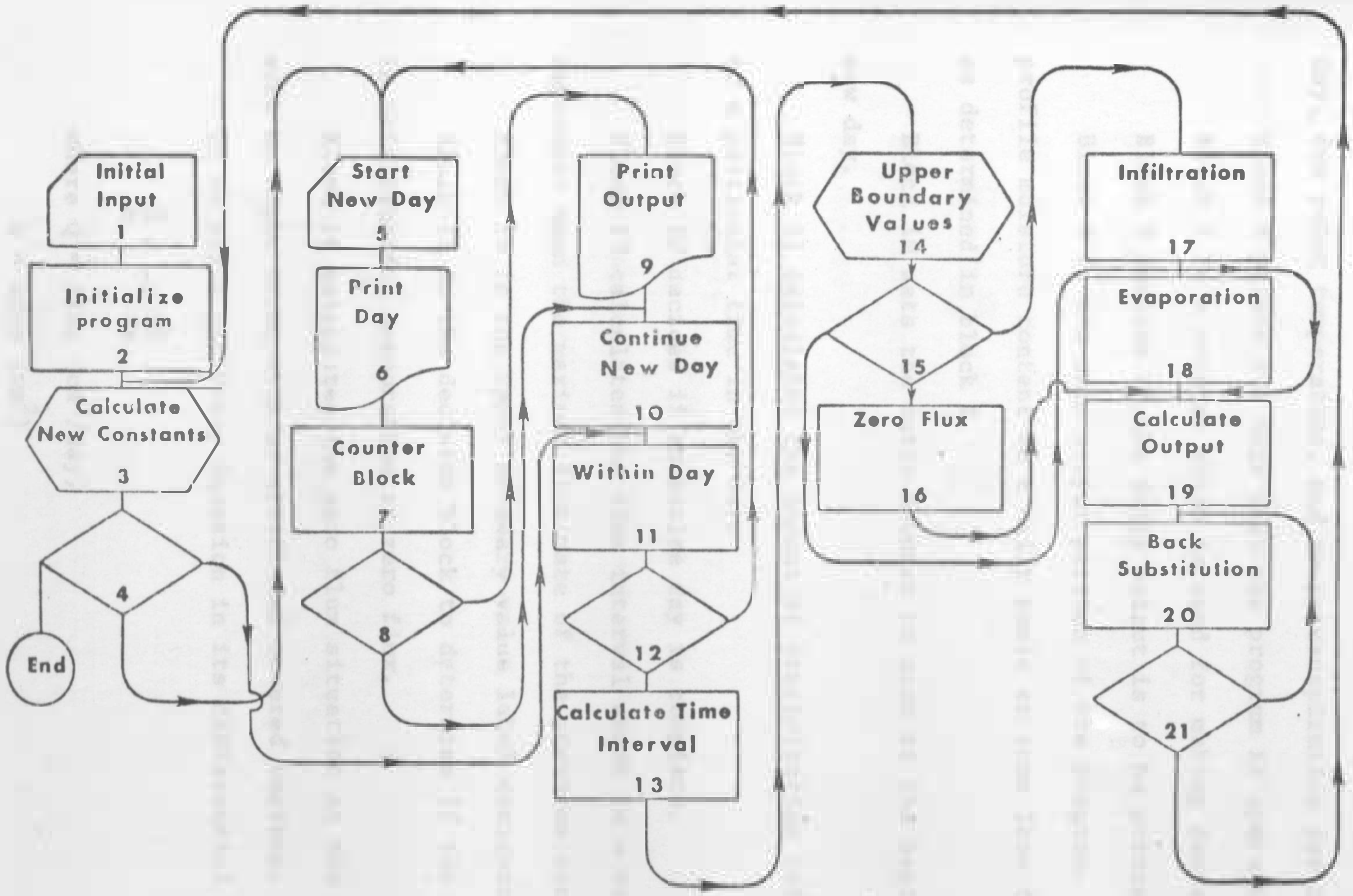


Figure III-3. Flow diagram of simulation program.

temperature, albedo of the field, daily precipitation, wind run for the day, dew point temperature, and evapotranspiration for the day.

Block 6 prints the date that the program is operating on.

Block 7 is a counter which is used for making decisions in block 8.

Block 8 decides if the daily output is to be printed.

Block 9 is the main output portion of the program. It prints the profile moisture content on a daily basis or some less frequent output as determined in block 8.

Block 10 sets the daily counter to zero at the beginning of a new day.

Block 11 calculates the amount of precipitation left to infiltrate at a particular time in the day.

Block 12 decides if an entire day is complete.

Block 13 calculates the time interval which is a variable and dependent upon the maximum flux rate of the previous cycle.

Block 14 is the upper boundary value label designator.

Block 15 is the decision block to determine if the upper boundary is infiltration, evaporation or zero flux.

Block 16 calculates the zero flux situation at the top boundary such as might occur with an artificial covered surface.

If we write the Darcy Equation in its differential form we have

$$\frac{Q}{A} = -K\,\frac{dh}{dz} \qquad [58]$$

where Q = flux (cm^3/day)

A = area (cm^2)

K = hydraulic conductivity (cm/day) = $K(\theta)$

h = total head (cm) = $(\Psi - z)$ where Ψ = soil moisture tension

z = depth (cm)

using diffusivity terms we can say

where $D = K \frac{d\Psi}{d\Theta}$

$$\frac{Q}{A} = -K \frac{d(\Psi-z)}{dz} = -\frac{d\Theta}{d\Psi} D\{\frac{d\Psi-dz}{dz}\} = -D \frac{d\Theta}{dz} + D \frac{d\Theta}{d\Psi} \qquad [59]$$

and setting $Q = 0$

$$\rightarrow D \frac{d\Theta}{d\Psi} = D \frac{d\Theta}{dz} \text{ or } K = D \frac{d\Theta}{dz} \qquad [60]$$

or we could write

$Kdz/D = d\Theta$

and writing $dz = \Delta z$ and $d\Theta = \Theta_{j+1}^{n+1/2} - \Theta_j^{n+1/2}$ [61]

$$\Theta_j^{n+1/2} = -K\Delta z/D + \Theta_j^{n+1/2}$$

If we assume that this fits any equation of the form $\Theta_j = E_j\Theta_{j+1}+F_j$

then $E_j = 1$ and $F_j = -K/D\ \Delta z$

Block 17 deals with the infiltration case.

At infiltration

$$\Theta_j = \Theta_{j+1} = \Theta sat \qquad [62]$$

to fit our arbitrary equation $\Theta_j = E_j\Theta_{j+1} + F_j$

$E_j = 1$

$\rightarrow F_j = 0$

or

$E_j = 0$

$\rightarrow F_j = \Theta sat$

but the first equation, although a solution, adds nothing to the system, so we utilize the second.

Block 18 is the block dealing with surface evaporation from an open soil profile.

Looking at the Darcy Equation we can again assume that

$$Q/A = -K \frac{dh}{dz} = -\text{evaporation (E)}$$

or $$E = K \frac{dh}{dz} = D \frac{d\Theta}{dz} - K$$

$$\left(\frac{E+K}{D}\right) \Delta z = \Theta_{j+1} - \Theta_j \qquad [63]$$

$$\Theta_j = \Theta_{j+1} - \Delta z\left(\frac{E+K}{D}\right) \qquad [64]$$

and to fit the arbitrary equation of the form

$$\Theta_j = E_j \Theta_{j+1} + F \qquad [65]$$

$\rightarrow$

$$E_j = 1 \text{ and } F_{j-1} = -\Delta z\left(\frac{E+K}{D}\right)$$

Block 19 is the implicit tridiagonal solution portions of the finite difference equations. The basic equation to be worked with is the second degree partial differential equation which is referred to as the basic flow equation with gravity considered. It is written

$$\frac{\partial\Theta}{\partial t} = \frac{\partial\{D(\Theta)\frac{\partial\Theta}{\partial z} - K(\Theta)\}}{\partial z} \qquad [66]$$

written in finite difference form at time n + 1/2 and depth j

$$\frac{\Theta_j^{n+1} - \Theta_j^n}{\Delta t} = \frac{1}{\Delta z}\{D(\Theta)_{j+1/2}^{n+1/2}\left(\frac{\Theta_{j+1}^{n+1} - \Theta_j^{n+1} + \Theta_{j+1}^n - \Theta_j^n}{2\Delta z}\right) \qquad [67]$$

$$-K(\Theta)_{j+1/2}^{n+1/2} - D(\Theta)_{j+1/2}^{n+1/2}\left(\frac{\Theta_j^{n+1} - \Theta_{j-1}^{n+1} + \Theta_j^n - \Theta_{j-1}^n}{2\Delta z}\right) + K(\Theta)_{j-1/2}^{n+1/2}\}$$

if we let

$$A = \frac{D(\Theta)^{n+1/2}_{j+1/2}\Delta t}{2\Delta z^2}, \quad C = \frac{D(\Theta)^{n+1/2}_{j-1/2}\Delta t}{2\Delta z^2}$$

$$B = 1+A+C \text{ and } D = \Theta^n_j(1-A-C)+A\Theta^n_{j+1} + C\Theta^n_{j-1} + (-K\Theta^{n+1/2}_{j+1/2}+K\Theta^{n+1/2}_{j-1/2})\frac{\Delta t}{\Delta z}$$

then we have the equation

$$A\Theta^{n+1}_{j+1} - B\Theta^{n+1}_j + C\Theta^{n+1}_{j-1} = D \qquad [68]$$

Now we have assumed in blocks 16, 17 and 18 that the upper boundary condition fits the form

$$\Theta^{n+1}_{j-1} = E_{j-1}\Theta^{n+1}_j + F_{j-1} \qquad [69]$$

and $$\Theta^{n+1}_j = E_j\Theta^{n+1}_{j+1} + F_j \qquad [70]$$

Taking these last three equations and combining them we have

$$E_j = A/(B-CE_{j-1}) \qquad [71]$$

$$F_j = (D+CF_{j-1})/(B-CE_{j-1}) \qquad [72]$$

Block 20 is the back substitution portion of the equations. After the entire profile of E_j and F_j values has been calculated these E_j+F_j are back substituted into the equation

$$\Theta^{n+1}_j = E_j\Theta^{n+1}_{j+1} + F_j \qquad [73]$$

where for the first Θ^{n+1}_{j+1}, the bottom boundary value is utilized for Θ^{n+1}_{j+1} and the entire profile of Θ^{n+1}_j can be calculated from the previous depth.

Block 21 returns the program to calculate new constants based upon the new thetas just calculated and continues the above procedure for a new time period.

CHAPTER III

SECTION 3

RESULTS AND DISCUSSION OF SOIL WATER MOVEMENT SIMULATION

Figure III-1 is an example of three of the nine Ψ vs Θ curves developed by Stone (1973) and Frankenstein (1973) compared with three curves at the same depth from the Ψ vs Θ subroutine. Although the curves generated by the subroutine are not in perfect agreement with the measured data, the maximum deviation of the dependent variable within the subroutine was 250 cm tension. This deviation is well within the variability of in-field measurements taken from several sites.

Experience has indicated that a linear exponential function should give good fit to Ψ vs Θ and K vs Θ field data. For this reason and for simplicity and brevity, we chose to use a series of linear exponential equations in our Ψ vs Θ and K vs Θ subroutines.

Within the water movement simulation program, the subroutines are accessed thousands of times within a monthly cycle. The use of a simple and brief procedure minimized computer time. It should be noted that the mathematical representation of the Ψ vs Θ data was computed to the 15,000 cm tension point in the subroutine but in Figure III-1 curves were plotted to only the 800 cm moisture tension point.

In the development of a mathematical representation of K vs Θ, three approaches were utilized before satisfactory results were obtained.

Initially Stone's K vs Θ values were fitted by a series of linear exponential equations. When utilized in the simulation procedure this method resulted in the continual problem of generating results which were orders of magnitude out of reality in the 90-110 cm depth. This problem may be attributed to the concave upward slope of the 90-110 cm K vs Θ curve as shown in a similar manner at the 70-90 cm depth in Figure III-2. Comparing this curve with the shape of K vs Θ curves at deeper and shallower depths and consideration of Ψ vs Θ for similar depths leads one to suspect the accuracy of the fitting of curves at this depth.

To extrapolate into regions where measured data were not available, the method of Green and Corey (1971) which calculates K vs Θ from the Ψ vs Θ was examined. This procedure utilizes a correction term or matching factor. Green and Corey recommend using the saturated hydraulic conductivity of the soil as the matching factor. For the Great Bend silt loam a saturated hydraulic conductivity matching factor of 20 cm/day was assumed for the entire profile. Using Green and Corey with a matching factor of 20 cm/day resulted in curves which, compared with curves in Figure III-2, were considerably to the right of all of the plotted curves. The moisture profile simulated using the saturated K of 20 cm/day appears wetter throughout than the moisture content found in a field.

The method which resulted in acceptable output was the Green and Corey procedure, using a matching factor taken near the middle of Stone's K vs Θ data. After the generation of a K vs Θ data set, the functional relationship for the subroutine was developed using a series of linear exponential equations. Figure III-2 shows three of these curves.

The only valid test of any theoretical simulation procedure is whether or not the procedure can duplicate a real situation. For the period represented by Julian date 6298 (24 Oct., 1976), to 6323 (18 Nov., 1976) a drainage plot of Great Bend silt loam located on the James Valley Research and Extension Center was monitored and simulated. Initially it was intended to simulate the field conditions for the entire winter period but after 6323 (18 Nov., 1976) both the simulation procedure and the field data remained constant.

In our first attempt at simulating drainage, we followed recommendations of laboratory studies (Liakopoulos, 1964) conducted using homogenous profiles, therefore we assumed an interval between nodes of about 5 cm. When initial saturated profile data was loaded into the simulation program a great deal of instability was found at the top of the profile which resulted in unrealistic soil profiles. After trying many different procedures to correct this problem we reduced the node interval at the top of the profile to 0.5 cm and used a varied interval throughout the rest of the profile. The results are shown in Figure III-4. As the calculations proceeded with time, there appears to be considerably more stability throughout the entire profile than was found using the constant 5 cm nodal system.

On 6298, (24 Oct., 1976) three plots of Great Bend silt loam were irrigated to use for verification of the drainage simulation procedure. Figures III-4, III-5 and III-6 represent soil moisture profiles on

Figure III-4. Moisture profile on day 6299.

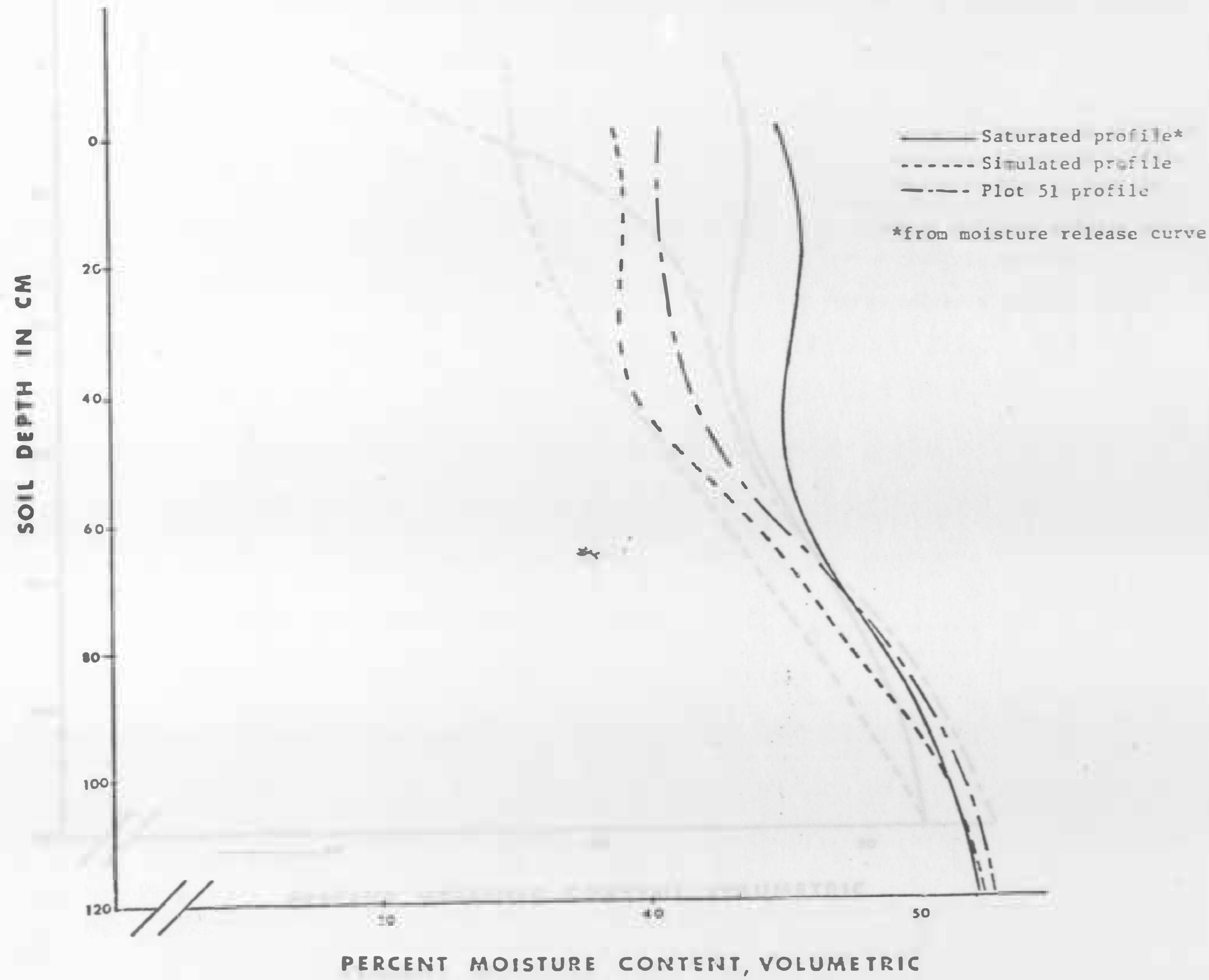


Figure III-5. Moisture profile on day 6313.

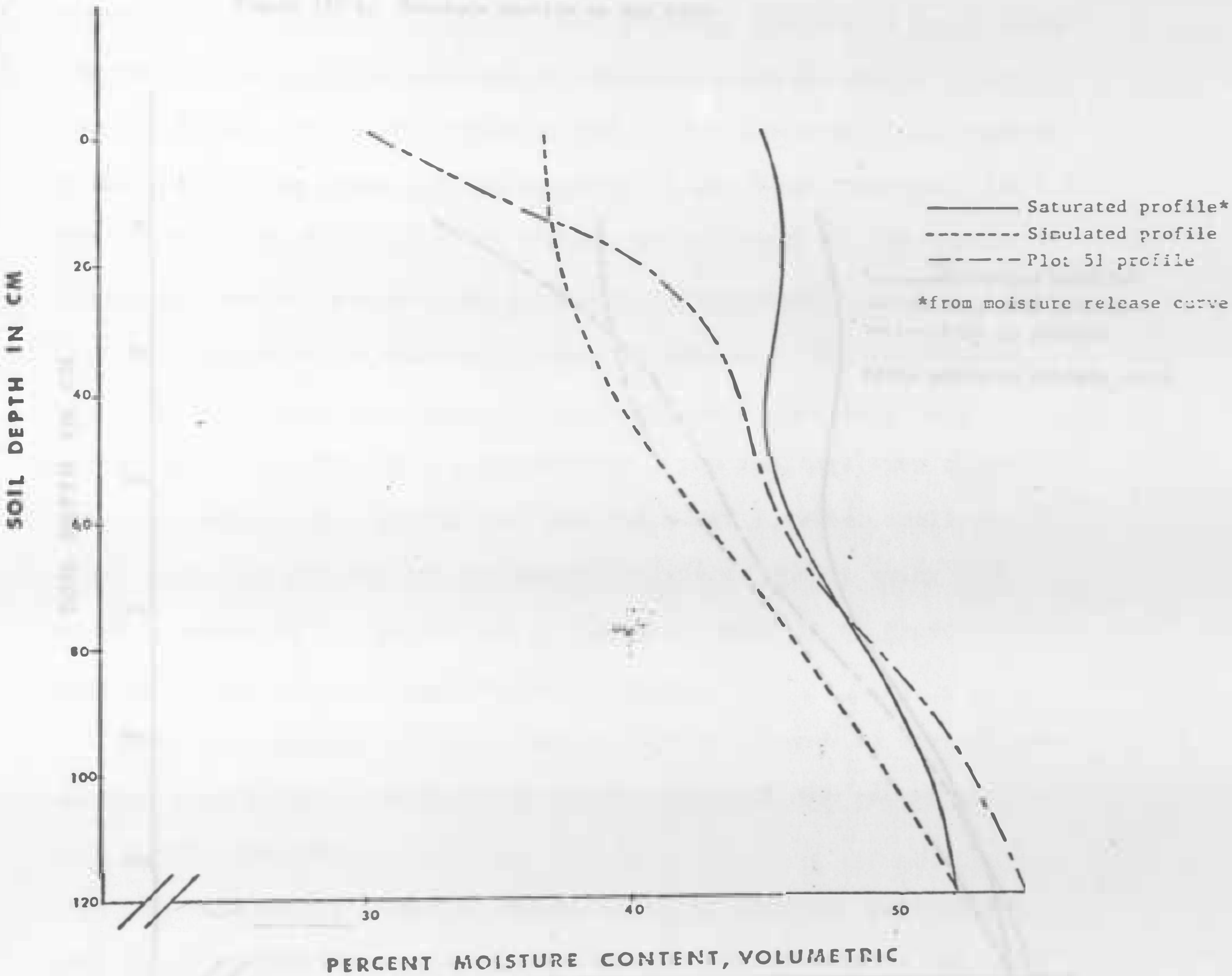
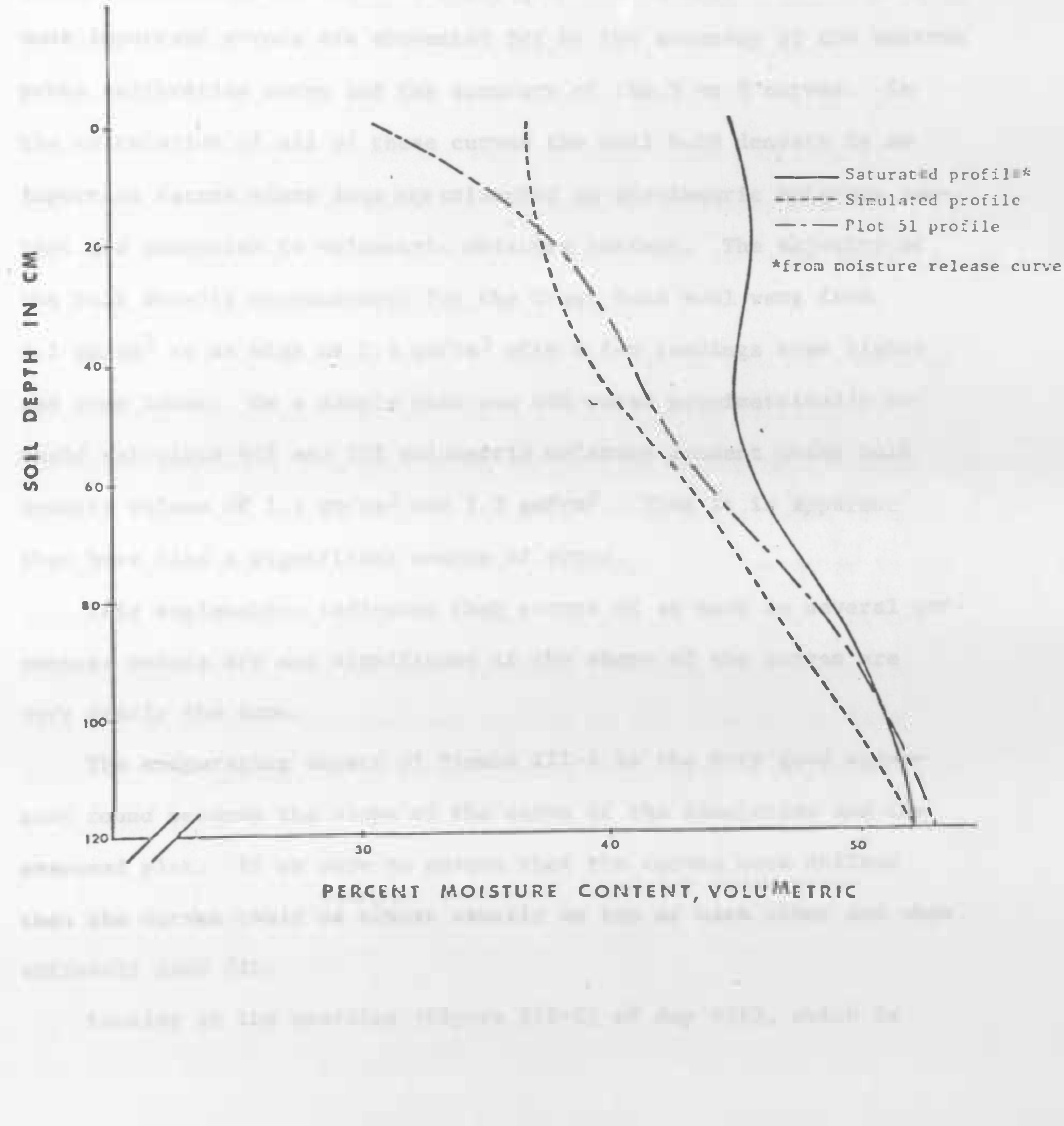


Figure III-6. Moisture profile on day 6323.



dates 6299, 6313, and 6323. Included in these figures are a plot of the computer simulated profile, a field measured profile and the saturated profile obtained from extrapolation of the moisture release curve to zero tension. There are many possible sources of error which can affect the apparent accuracy of the simulation procedure. The most important errors are accounted for by the accuracy of the neutron probe calibration curve and the accuracy of the Ψ vs Θ curves. In the calculation of all of these curves the soil bulk density is an important factor since data are collected as gravimetric moisture content and converted to volumetric moisture content. The majority of the bulk density measurements for the Great Bend soil vary from 1.1 gm/cm^3 to as high as 1.3 gm/cm^3 with a few readings even higher and some lower. On a sample that was 40% water gravimetrically we would calculate 44% and 52% volumetric moisture content using bulk density values of 1.1 gm/cm^3 and 1.3 gm/cm^3. Thus it is apparent that here lies a significant source of error.

This explanation indicates that errors of as much as several percentage points are not significant if the shape of the curves are very nearly the same.

The encouraging aspect of Figure III-4 is the very good agreement found between the slope of the curve of the simulation and the measured plot. If we were to assume that the curves were shifted then the curves could be almost exactly on top of each other and show extremely good fit.

Looking at the profiles (Figure III-5) of day 6313, which is two weeks later, we see that the deviation in the profiles is greater than early in the drainage cycle. We find that the surface is considerably drier than the simulation procedure predicts. There are two possible explanations for this result. The first and most obvious is that the simulation procedure is not working within the depth of the top 40 cm of the surface drainage model. The second possible explanation is that the plastic tarp used to cover the top of the plots was not creating a surface seal and significant amounts of water were lost from the surface by evaporation. Evaporation off the surface would be a contradiction of one of our basic assumptions, i.e. that the surface boundary has zero flux.

Except at the surface there is good similarity between the shape of curves of the simulation profile and the measured profile. In the Great Bend silt loam soil there are less permeable layers which could cause a temporary water table to build up. This would cause results similar to what is seen here. Had a piezometer been installed in the plots, this assumption could have been refuted or verified. On day 6323, Figure III-6, the measured profiles appear to have about the same deviation from the simulated profile as they did on the first day after irrigation. This would further substantiate the supposition of a false water table.

SUMMARY AND CONCLUSION

In the first chapter we discussed winter conservation of fall applied irrigation water and cultural practices which result in increased spring moisture content. We defined a percent loss (PL) term and developed, with a statistical procedure, a functional relationship for use in the determination of how much fall irrigation water can feasibly be applied. A simple predictive snow accumulating function was developed and statistical constants for the function were generated. Although we think that the methodology presented in Chapter I for studying overwinter moisture loss is valid and functional, to improve upon our work it is necessary to look at more locations and additional years of data. Improvements in our PL function might include consideration of the interaction of some of the independent variables. Within the Great Plains Region the tremendous variability in overwinter precipitation precludes the need for a more complex theoretical prediction equation which would use precipitation as an independent variable.

The basic concept of using an analytical procedure to determine when to fall irrigate and how much water to apply appears considerably more desirable than does the "by guess or by golly" method which is the only alternative today.

In order to compute the monetary value of a specific fall irrigation (assuming that the PL function is known) or a particular irrigation at any time during the growing season, the development of a crop production function is necessary. Chapter II was devoted to the development of two corn production functions based upon six physiological stages of maturity and the accumulative soil water deficit within each stage. One model assumed that the function was additive between stages while the second model assumed the function to be multiplicative between stages. No conclusion was reached as to which model predicted yields most accurately. Again, more data are needed to develop confidence in the statistical constants of the regression analysis.

When considering chapters I and II simultaneously, we conclude that full soil water profiles early in the season combined with early season rains can result in yield depression.

A crop production function based upon water availability at a particular physiological stage of maturity must be used to objectively maximize production while minimizing water and energy consumption.

In Chapter III we simulated the drainage of water from a field profile by solving the theoretical flow equation.

We have confidence that this method of solution to the basic flow equation produces realistic results when the supporting subroutines describing the soil parameters and the boundary conditions are accurate. Immediate improvements to the simulation procedures will come in the form of increased accuracy from these supporting routines. It is the feeling of most soil scientists working with soil water simulation procedures that the greatest error in simulation procedures comes as a result of our inability to accurately determine realistic Ψ vs Θ and K vs Θ values. Unfortunately the majority of Ψ vs Θ curves used are determined in the laboratory from field cores. A more desirable method would be to develop a simple but accurate in situ method for finding Ψ vs θ curves.

In the future we see simulation procedures applied to field problems. One example where simulation procedures would be particularly useful is in the study of root extraction patterns. If simulation of a profile without roots is possible then changes in moisture content which depart from predicted movement could be attributed to rooting patterns and root moisture extraction.

A second problem where simulation procedures can be applied in years to come is the problem of improving salt leaching efficiency from a given amount of irrigation water. This work would deal with specific ion equilibria between the soil complex, and the soil water solution.

A third area where simulation may be of value is in the study of the effect of the sodium ion upon the hydraulic conductivity of a soil profile.

Although simulation is not a cure-all, it is possible that the addition of theoretical simulation can be of considerable value in our quest to better understand soil-water phenomena.

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